# A TEXT BOOK OF
# PRINCIPLES OF ELECTRICAL ENGINEERING

## ET 125

## DAE FIRST YEAR ELECTRICAL TECHNOLOGY

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# PREFACE

We are thankful to Almighty ALLAH who gave us an opportunity to write the book, named ‘Principles of Electrical Engineering’ as Textbook of Diploma of Associate Engineer (DAE) 1st Year Electrical Technology, intending to cover the new syllabus. Throughout the book, emphasis has been given over general and specific objectives of course, easy approach to transfer knowledge, transposition of formulae, and logical layout of solutions of numerical problems, neatness and clarity of arrangement of material, systematic use of all relevant pictures, circuit diagrams, Phasor and vector diagrams. This book also includes objective type, short and long questions at the end of each chapter.

This book covers topics including understanding electricity, established laws and concepts, and their application in different situations. Thus, solving problems also forms part of the cognition of these concepts. This book aims at providing a strong foundation in these basic concepts and laws of electricity, along with an appreciation of the magnitudes of the quantities involved or to be guessed, through solving numerical problems. The concepts will be further strengthened through extensive Laboratory work. Normally students face difficulty in solving complicated problems because they do not make a systematic attempt. We have attempted to help the students to overcome the difficulty by providing detailed instructions for an orderly approach. Difficult procedures and types of problems appearing in the exercise are illustrated by carefully explained examples. In the presentation of these illustrated examples, we have avoided unnecessary explanations. It is hoped that this book will help to give students a good foundation in old and new techniques.

We would like to express, sincere and thanks to Engr. Abdul Wasay General Manager Academics, and Engr. Syed Muhammad Waqar Ud- Din Manager (Curriculum) Section Academics Wing, who took keen interest and inspired us for the completion of this task. We made every effort to make the book valuable both for students and teachers; however, we shall gratefully welcome to receive any suggestion for the further improvement of the book.

**AUTHORS**

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| 9.13 | Calculation of P.F with active & reactive power. |
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## ET 125 PRINCIPLES OF ELECTRICAL ENGINEERING

| **Total Contact Hours:** | | **T** | **P** | **C** |
|---|---|---|---|---|
| | | 3 | 6 | 5 |
| Theory: | 96 | Practical's: | 192 | |

**AIMS** Understanding electricity involves the sound familiarity with the established laws and concepts, and their application in different situations. Thus, solving problems also forms part of the cognition of these concepts.

This course aims at providing a strong foundation in these basic concepts and laws of electricity, along with an appreciation of the magnitudes of the quantities involved or to be guessed, through solving numerical problems. The concepts are further strengthened through extensive Laboratory work.

### COURSE CONTENTS

### UNIT-I D.C. FUNDAMENTAL

### 1. BASIC CONCEPTS OF ELECTRICITY (15 Hrs.)

1.1 Electron theory.

1.2 Electric Charge, Electric Current, Potential Difference, Resistance, Conductance.

1.3 Conductor, Insulator, Semiconductor.

1.4 Ohm's Law.

1.5 Laws of resistance

1.6 Effect of temperature on resistance.

1.7 Temperature coefficient of resistance.

1.8 Series and parallel circuits

1.9 Resistances in series and parallel.

1.10 Division of voltage in series circuit.

1.11 Division of current in parallel circuit.

1.12 Resistances in combination circuits.

**2. NETWORK THEOREMS (5 Hrs.)**

2.1 Active & passive circuits, node, branch, mesh and loop in Electrical circuits.

2.2 Kirchhoff's law I - current law.

2.3 Kirchhoff's law II-voltage law.

2.4 Application of Kirchhoff's laws.

2.5 Problem solving with Kirchhoff's Laws in D.C. circuit. (Simple problems)

2.6 Superposition theorem.

2.7 Maximum power transfer theorem.

2.8 Thevenin's theorem.

**3. WORK, POWER AND ENERGY (8 Hrs.)**

3.1 Work, Power and Energy

3.2 Conversion of electrical power into mechanical power.

3.3 Energy billing.

3.4 Heating effect of current.

3.5 Joule's Law.

3.6 Thermal efficiency.

**4. ELECTROMAGNETISM (9 Hrs.)**

4.1 Magnet and magnetism.

4.2 Basic concepts and terminology of magnetism.

4.3 Absolute and relative permeability.

4.4 Magnetic hysteresis

4.5 Laws of magnetic force.

4.6 Magnetic field due to a straight current carrying conductor.

4.7 Right hand thumb rule, Cork-Screw rule.

4.8 Magnetic field of coil

4.9 Right hand gripping rule, End rule

4.10 Effect of iron core in a coil.

4.11 Mechanical force on a current carrying conductor in a magnetic field.

4.12 Fleming's right hand and left-hand rules.

**5. ELECTROMAGNETIC INDUCTION (3 Hrs.)**

5.1 Faraday's Laws of electromagnetic induction.

5.2 Dynamically and statically induced EMF.

5.3 Lenz's Law.

5.4 Self and Mutual induction.

5.5 Eddy current.

**6. ELECTROSTATICS (8 Hrs.)**

6.1 Static Electricity.

6.2 Basic concepts and terminology of Electrostatics.

6.3 Absolute and relative permeability of a medium.

6.4 Laws of Electrostatics.

6.5 Capacitor and its types.

6.6 Capacitance

6.7 Capacitors in series and parallel.

6.8 Charging of a capacitor and its equation.

6.9 Discharging of a capacitor and its equation.

**UNIT-II A.C FUNDAMENTALS**

**7. FUNDAMENTALS OF A.C (10 Hrs.)**

7.1 Alternating current & voltage.

7.2 Working principle of Alternating current generator.

7.3 Simple loop Alternator, Relationship between Speed, poles and frequency.

7.4 Sinusoidal EMF Equation.

7.5 Wave forms and types

7.6 Cycle, Time Period, Frequency, Maximum (Peak) value, Peak to Peak value, Instantaneous value, Average value, R.M.S value (Effective value), Form factor, Peak factor.

7.7 Representation of A.C. through vectors.

7.8 Phase, Phase difference, In-phase, Out of Phase, Phase Quadrature, Anti-phase, Lagging & Leading.

7.9 Phasor Diagrams.

7.10 Complex Numbers.

7.11 Rectangular and Polar form conversion

**8. SINGLE PHASE A.C. CIRCUIT (20 Hrs.)**

8.1 A.C through pure Resistance and Vector Diagram.

8.2 A.C through pure Inductance and Vector Diagram.

8.3 A.C through pure Capacitance and Vector Diagram.

8.4 A.C through pure Resistance & Inductance in series, including wave forms and Phasor diagram.

8.5 A.C through Resistance and Capacitance connected in series including wave forms and phasor diagram.

8.6 Voltage, current and power relation in A.C. R-L and R-C Circuits.

8.7 R.L.C series circuit.

8.8 Impedance Triangle, Phase angle.

8.9 Active and Reactive component. Actual power, Apparent Power, Reactive Power, relationship, Power triangle, Power factor

8.10 Parallel A.C circuits.

8.11 Solution of Parallel circuits by vector and admittance method.

8.12 Solution of simple problems with Phasor Algebra.

8.13 Power factor improvement with static capacitor.

8.14 Resonance circuit.

8.15 Series, parallel resonance circuit.

8.16 Problems on resonant circuits.

8.17 Harmonics, True Power Factor

**UNIT-III POLY-PHASE FUNDAMENTALS**

**9. POLY-PHASE A.C. CIRCUIT (18 Hrs.)**

9.1 Generation of two phase, three Phase EMF.

9.2 Advantages of Poly Phase system.

9.3 Star & Delta connections.

9.4 Relationship between line and phase values of star & delta connections.

9.5 Comparison of Star and Delta connections, their uses, and conversion.

9.6 Power in a three-phase balanced load.

9.7 Vector diagram of Star and Delta connected load.

9.8 Current in Neutral in a 3 Phase circuit.

9.9 Problem solving on star and delta connections.

9.10 Measurement of power with one-watt meter without the use of Neutral wire.

9.11 Measurement of power with two-watt meters and its vector diagram.

9.12 Measurement of Power with three-watt meters and its vector diagram.

9.13 Measurement of Reactive power in a three-phase circuit.

9.14 Calculation of P.F. with Active and reactive power.

9.15 Phase sequence.

9.16 Advantages of 3 Phase supply over single Phase supply.

9.17 Problem solving on 3 Phase circuits.

## ET- 125 PRINCIPLES OF ELECTRICAL ENGINEERING

## INSTRUCTIONAL OBJECTIVES

## UNIT-I: D.C. FUNDAMENTALS

**1. UNDERSTAND BASIC CONCEPTS OF ELECTRICITY**

1.1 State Electron theory.

1.2 Define Electric Charge, Electric Current, Potential Difference, Resistance, and Conductance and state their units

1.3 Define and compare Conductor, Insulator & Semi-conductor.

1.4 State Ohm`s law and calculations.

1.5 Explain laws of resistance and calculations.

1.6 State effects of temperature on Resistance.

1.7 Calculate temperature co-efficient of Resistance.

1.8 Define series and parallel circuits of resistances with their properties.

1.9 Determine total resistances in series & parallel circuits.

1.10 Calculate division of voltage in series circuits.

1.11 Calculate division of current in parallel circuits.

1.12 Draw equivalent circuits for combination of resistances, calculate equivalent resistance.

**2. UNDERSTAND KIRCHHOFF'S LAWS**

2.1 Define active circuit, passive circuit, node, branch, mesh & loop.

2.2 State Kirchhoff's1st Law - (current Law).

2.3 State Kirchhoff's 2ndLaw - (voltage Law).

2.4 Give examples for applications of Kirchhoff's Laws.

2.5 Solve simple problems on Kirchhoff's Laws in D.C circuits.

2.6 State superposition theorem.

2.7 State maximum power transfer theorem.

2.8 State Thevenin's Theorem.

## 3. UNDERSTAND WORK, POWER & ENERGY

3.1 Define work, electrical power, mechanical power and energy with their units. Calculations on Power and Energy

3.2 State formula for conversion of Electrical Power (Watt) to Mechanical Power (H.P).

3.3 Calculate Energy billing of an installation.

3.4 Explain heating effect of current.

3.5 State Joule's Law of current.

3.6 Define thermal efficiency. Solve problems on Thermal Efficiency.

## 4. UNDERSTAND MAGNETIC EFFECTS OF ELECTRIC CURRENT

4.1 Define magnet and magnetism, types of magnet with properties.

4.2 Define Magnetic field, Define Magnetic lines of force and properties, Define Magnetic flux and state unit, Define Magnetic flux density and state unit, Define Magneto-motive force and state unit, Define Reluctance and Permeance, Define Magnetic field strength (magnetizing force) and Retentivity, Define Magnetic circuit, Compare magnetic circuit with electric circuit.

4.3 Define Absolute & Relative permeability.

4.4 Define magnetic hysteresis, state magnetization curve (B – H curve).

4.5 Explain Laws of Magnetic force.

4.6 Describe Magnetic field of a straight current carrying conductor.

4.7 State right hand thumb rule, State cork-screw rule.

4.8 Determine Magnetic field of a coil.

4.9 State Right hand gripping rule, State End rule

4.10 Describe effect of iron core in a coil.

4.11 Explain mechanical force on a current carrying conductor in a magnetic field.

4.12 State Fleming's Right hand & Left-hand rules.

**5. UNDERSTAND ELECTROMAGNETIC INDUCTION**

5.1 State Faraday's Laws of Electromagnetic Induction.

5.2 State dynamically & statically induced EMF.

5.3 Explain Lenz's Law

5.4 State self & mutual inductance. State unit of inductance.

5.5 State Eddy current.

**6. UNDERSTAND FUNDAMENTALS OF ELECTROSTATICS**

6.1 Define static-electricity (Electrostatics).

6.2 Define Electric field, Define Electric lines of force and properties, Define Electric flux, Electric flux density, Electric field strength

6.3 Describe Absolute & Relative Permeability of a Medium.

6.4 State Laws of Electrostatics.

6.5 Define capacitor, list types of capacitors.

6.6 State the term capacitance and state its unit.

6.7 Define series and parallel circuits of capacitors with their properties, Solve problems on capacitors in series & parallel.

6.8 Explain charging of capacitor along with equation.

6.9 Explain discharging of capacitor along with equation.

**UNIT-II: A.C. FUNDAMENTALS**

**7. UNDERSTAND A.C. FUNDAMENTALS**

7.1 Define alternating current & voltage.

7.2 Describe principle of working of A.C. Generator.

7.3 Explain simple loop Alternator & relationship between speed, poles and frequency.

7.4 State sinusoidal E.M.F. equation.

7.5 Define Wave form, State types; Sinusoidal wave form and non-sinusoidal wave forms; square, triangular, saw-tooth.

7.6 Define terms Cycle, Time Period, Frequency, Maximum (Peak) value, Peak to Peak value, Instantaneous value, Average value, R.M.S value (Effective value), Form factor, Peak factor, Phase, Phase difference, In-phase, Out of Phase, Phase Quadrature, Lagging & Leading waves.

7.7 Explain how AC quantities can be represented by vectors.

7.8 Define the terms according to wave form and vectors; Phase, Phase difference, In-phase, Out of Phase, Phase Quadrature, Anti-phase, Lagging & Leading.

7.9 Draw phasor diagrams.

7.10 Define Complex number; describe rectangular form and polar form of A.C quantities.

7.11 Conversion from R-P form and P-R form, simple calculations.

**8. UNDERSTAND A.C. CIRCUITS (SINGLE PHASE)**

8.1 Explain the effects of A.C. supply through pure resistance, inductance & Capacitance with their waveforms, vector diagrams and power curves.

8.2 Describe the effects of A.C. supply through RL and RC Series circuits with the help of waveforms and vector diagrams.

8.3 Derive voltage current & power relation in A.C. circuits.

8.4 Explain R.L.C series circuit and Solve examples on R.L.C. series circuit.

8.5 Define terms impedance, impedance triangle. Draw impedance triangle of A.C Circuits, phase angle.

8.6 Describe active & reactive component, actual power, apparent power& reactive power with relationships, Define Power factor

8.7 Explain parallel A.C circuits (R.L.C).

8.8 Solve problems on parallel A.C. circuits

8.9 Explain power factors improvement with static capacitor bank.

8.10 Write relationship for V.I.Z. for resonance circuit in series & parallel.

8.11 Solve simple problem on resonance circuits.

8.12 State harmonics.

8.13 Concept of True power factor

**UNIT-III: POLYPHASE FUNDAMENTALS.**

**9. UNDERSTAND POLYPHASE A.C. CIRCUITS**

9.1 Explain generation of two-phase & 3-phase e.m.f.

9.2 Explain advantages of A.C. polyphase system.

9.3 Draw & explain star & delta connections.

9.4 Calculate relationship between line & phase values in star & delta.

9.5 Compare star & delta connections with their uses.

9.6 State power equation for 3-phase system.

9.7 Draw vector diagrams of star & delta connected loads.

9.8 State value of current in neutral in a 3-phase balanced circuit.

9.9 Solve problems on star and delta connected 3-phase balanced load.

9.10 Explain Measurements of power with one wattmeter without the use of neutral wire.

9.11 Describe Measurement of power with two-watt meters along with its vector diagram.

9.12 Calculate power with three watt-meters along with vector diagrams.

9.13 Describe Measurement of Reactive power in a three-phase circuit.

9.14 Solve problems on P.F with active & reactive power.

9.15 Define phase sequence, Explain phase sequence meter.

9.16 Explain advantages of 3-phase supply over single phase supply.

9.17 Solve problems on 3-phase circuits. (Balanced load)

# DC FUNDAMENTALS
# PAPER-A

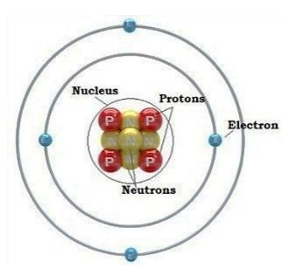
## Chapter # 1

## BASIC CONCEPTS OF ELECTRICITY

### 1.1 ELECTRON THEORY:

According to the modem electron theory, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the neutron, the proton and the electron. The proton is defined as positively charged while the electron is defined as negatively charged. The neutron is uncharged i.e. neutral in nature possessing no charge. The mass of neutron and proton is same while the electron is very light, almost 1/1840th the mass of the neutron and proton. The following table gives information about these three particles.

| Fundamental particles of matter | Symbol | Nature of charge Possessed | Mass in kg. |
|---|---|---|---|
| Neutron | N | 0 | $1.675 X10^{-27}$ |
| Proton | P | + | $1.675x10\text{-}^{27}$ |
| Electron | E | – | $9.107X\ 10^{-31}$ |




### STRUCTURE OF AN ATOM:

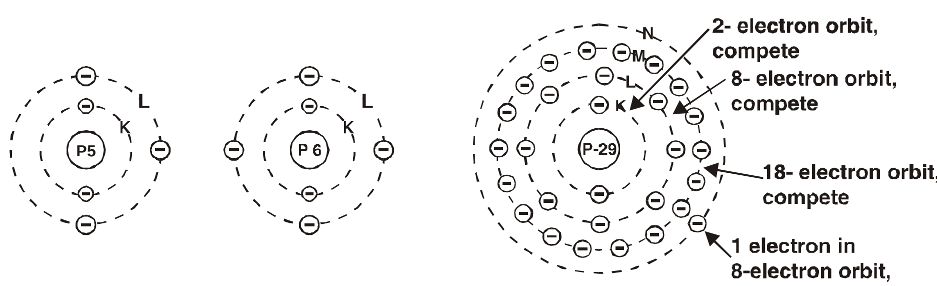
All of the protons and neutrons are bound together into a compact nucleus. Nucleus may be thought of as a central sun, about which electrons revolve in a particular fashion. This structure surrounding the nucleus is referred as the electron cloud. In the normal atom the number of protons equal to the number of electrons. An atom as a whole is electrically neutral. The electrons are arranged in different orbits. The nucleus exerts a force of attraction on the revolving electrons and holds them together. All these different orbits are called shells and possess certain energy. Hence these are also called energy shells or quanta. The orbit which is closest to the nucleus is always under the tremendous force of attraction while the orbit which is farthest from the nucleus is under very weak force of attraction. In some atoms such valence electrons are so loosely bound to the nucleus that at room temperature the additional energy imparted to the valence electrons causes them to escape from the shell and exist as free electrons. Such free electrons are basically responsible for the flow of electric current through metals. The electrons which are revolving round the nucleus not revolve in a single orbit. Each orbit consists of fixed number of electrons. In general, an orbit can contain a maximum of $2n^2$ electrons where n is the number of orbits. So, first orbit or shell can occupy maximum of $2 \times 1^2$ i.e. 2 electrons while the second shell can occupy maximum of $2 \times 2^2$ i.e. 8 electrons and so on. The exception to this rule is that the valence shell can occupy maximum 8 electrons irrespective of its number. Let see the structure of different atoms.

**Examples:**

(i) Carbon has an atomic number 5 as it has 5 electrons. Distribution of electrons will be such that in first K shell there are 2 electrons and in second L shell there are 3 electrons.

(ii) Carbon has an atomic number 6 as it has 6 electrons. Distribution of electrons will be such that in first K shell there are 2 electrons and in second L shell there are 4 electrons.

(iii) Copper atom has 29 electrons. Distribution of electron in it will be such that in first shell 2 electrons, in second shell 8 electrons, in third shell 18 electrons and in fourth shell there is 1 electron. It is an unstable atom.




(i) Boron Atom (ii) Carbon Atom (iii) Copper Atom

### 1.1.1 Valance Electrons:

An outer shell of an atom is called a valance shell and revolving in it are called Valance Electrons.

## 1.2 (a) CONDUCTOR:

Conductor is a material that permits the flow of electrical current in one or more directions. For example, a wire is an electrical conductor that can carry electricity along its length. In metals copper or aluminum are vastly used as conductor. In conductors the movable charged particles are electrons.

Usually metals are good conductors e.g. Silver, Copper, Aluminum, Brass, Iron and Nickle etc.

Silver is more conductive than copper, but due to cost it is not practical in most cases. However, it is used in specialized equipment, such as satellites, and as a thin plating to mitigate skin effect losses at high frequencies.

## 1.2 (b) INSULATOR:

An electrical insulator is a material which has much resistance to flow of free electron i.e. current. In outer most shell of these materials, number of electrons are more than four. There is a greater attraction of nucleus on them and do not leave their atoms easily e.g. mica, plastic, varnish, rubber, glass, Dry wood and dry thread etc.

## 1.2 (c) SEMICONDUCTOR:

Such materials, whose electric characteristics are between conductors and insulators, are called Semi-Conductors. In their outer most shell number of electrons is always 4. e.g. carbon, germanium and Silicon.

## 1.3 (a) RESISTANCE:

The electrical resistance of an electrical conductor is the opposition to the passage of an electric current through that conductor. The SI unit of electrical resistance is the ohm (Ω) and is denoted with the letter R.

The concept of resistance is analogous to the friction involved in the mechanical motion. Every metal has a tendency to oppose the flow of current. Higher the availability of the free electrons, lesser will be the opposition to the flow of current. The conductor due to the high number of free electrons offers less resistance to the flow of current. When the flow of electrons is established in the metal, the ions get formed which are charged particles as discussed earlier. Now free electrons are moving in specific direction when connected to external source of emf so such ions always become obstruction for the flowing electrons. So, there is collision between ions and free flowing electrons. This not only reduces the speed of electrons but also produces the heat. The effect of this is nothing but the reduction of flow of current. Thus, the material opposes the flow of current.

**Ohm:** If 1 ampere current passes through a conductor under 1-volt pressure then its resistance will be 1 ohm.

1 ohm = 1 Volt / 1 Amp.

## 1.3 (b) CONDUCTANCE:

The inverse quantity of resistance is called conductance, it is the indication of ease with which an electric current pass through material. Conductance is measured in Siemens (S) or mho (℧). It is denoted with letter 'G'.

$$G = 1 / R$$

## 1.3 (c) CONCEPT OF CHARGE:

In all the atoms, there exists number of electrons which are very loosely bound to its nucleus. Such electrons are free to wonder about, through the space under the influence of specific forces. Now when such electrons are removed from an atom it becomes positively charged. This is because of losing negatively charged particles i.e. electrons from it. As against this, if excess electrons are added to the atom it becomes negatively charged.

The following table shows the different particles and charge possessed by them.

| Particle | Charge possessed In Coulomb | Nature |
|---|---|---|
| Neutron | 0 | Neutral |
| Proton | $1.602 x 10^{-19}$ | Positive |
| Electron | $1.602 x 10^{-19}$ | Negative |

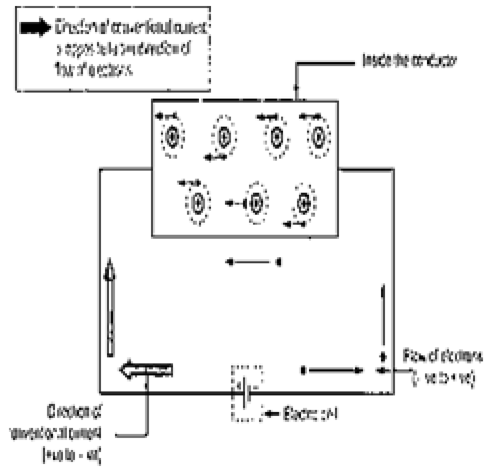
## 1.3 (d) UNIT OF CHARGE:

As seen from the above table that the charge possessed by the electron is very small hence it is not convenient to take it as the unit of charge. The unit of the measurement of the charge is Coulomb. The charge on one electron is $1.602x10^{-19}$, so one coulomb charge is defined as the charge possessed by total number of ($1/ 1.602x10^{-19}$) electrons i.e. $6.24x10^{18}$ number of electrons. Thus, 1 coulomb = charge on $6.24 x10^{18}$ electrons.

From the above discussion it is clear that if an element has a positive charge of one coulomb then that element has a deficiency of $6.24x10^{18}$numbers of electrons.

## 1.4 CONCEPT OF CURRENT:

It has been mentioned earlier that the free electrons are responsible for the flow of electric current. Let us see how it happens. To understand this, first we will see the enlarged view of the inside of a piece of a conductor. A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions inside the piece of a conductor.



***The flow of current***

The small electrical effort, externally applied to such conductor, makes all such free electrons, to drift along the metal in a definite particular direction. This direction depends on how the external electrical effort is applied to the conductor. Such an electrical effort may be an electrical cell, connected across the two ends of a conductor. The metal consists of particles which are charged. The like charges repel while unlike charges attract each other. But as external electric effort is applied, the free electrons as are negatively charged get attracted by positive of the cell connected.

And this is the reason why electrons get aligned in one particular direction under the influence of an electromotive force. Atoms, when they lose or gain electrons, become charged accordingly and are called ions. Now when free electron gets dragged towards positive from an atom it becomes positively charged ion. Such positive ion drags a free electron from the next atom. This process repeats from atom to atom along the conductor. So, there is flow of electrons from negative to positive of the cell, externally through the conductor across which the cell is connected. This movement of electrons is called an Electric current. The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This is called direction of conventional current. We are going to follow direction of the conventional current throughout this book i.e. from positive to negative terminal; of the battery through the external circuit.

## RELATION BETWEEN CHARGE AND CURRENT:

The current is flow of electrons. Thus, current can be measured by measuring how many electrons are passing through material per second. This can be expressed in terms of the charge carried by those electrons in the material per second. So, the flow of charge per unit time is used to quantify an electric current. The charge is indicated by Q coulombs while current is indicated by I. The unit for the current is Amperes which is nothing but coulombs/sec. Hence mathematically we can write the relation between the charge (Q) and the electric current (I) as,

$$\boxed{I = {}^{Q}/t}$$

| | | | |
|---|---|---|---|
| Where | I | = | Average current flowing |
| | Q | = | Total charge transferred |
| | t | = | Time required for transfer of charge |

## DEFINITION OF AMPERE:

A current of 1Ampere is said to be flowing in the conductor when a charge of one coulomb is passing through any given point in one second. Now 1 coulomb is $6.24 \times 10^{18}$ numbers of electrons. So, 1 ampere current flow means flow of $6.24 \times 10^{18}$ electrons per second across a section taken anywhere in the circuit.

## 1.5 ELECTRIC POTENTIAL / POTENTIAL DIFFERECE:

When two similarly charged particles are brought near, they try to repel each other while dissimilar charges attract each other. This means, every charged particle has a tendency to do work. The electric potential at a point due to a charge is one volt if one joule of work is done in bringing a unit positive charge.

Let us define now the potential difference. It is well known that; flow of water is always from higher level to lower level; flow of heat is always from a body at higher temperature to a body at lower temperature. Such a level difference which causes flow of water, heat and so on also exists in electric circuits. In electric circuits flow of current is always from higher electric potential to lower electric potential. So, we can define potential difference as "The difference between the electric potentials at any two given points in a circuit is known as Potential Difference This is also called voltage between the two points mid measured in volts. The symbol for voltage is V.

For example, let the electric potential of a charged particle A is say $V_1$ while the electric potential of a charged particle B is say $V_2$.Then the potential difference between the two particles A and B is $V_1 – V_2 = V$, If $V_1 – V_2 = V$ is positive we say that A is at higher potential than B while if $V_1 - V_2 = V$ is negative we say that B is at higher potential than A. As per the definition of volt, the V joules of work are to be performed to move unit charge from point of higher potential to point of lower potential. Thus, when such two points, which are at V, potential different are joined together with the help of wire, the electric current flows from higher potential to lower potential i.e. the electrons start flowing from lower potential to higher potential. Hence, to maintain the flow of electrons i.e. flow of electric current, there must be a potential.

## 1.6 OHM'S LAW:

The law was named after the German physicist Georg Ohm. Ohm's law states that;

1- The current in a circuit is directly proportional to the electric potential difference applied across its ends.

$$I \propto V$$

The greater the battery voltage (i.e., electric potential difference), the greater is the current, and the greater the resistance, the less the current. Charge flows at the greatest rates when the battery voltage is increased and the resistance is decreased. In fact, a twofold (twice) increase in the battery voltage would lead to a twofold increase in the current (if all other factors are kept equal).

2- The current in a circuit is inversely proportional to the total resistance offered by the external circuit.

$$I \propto \frac{1}{R}$$

An increase in the resistance of the load by a factor of two would cause the current to decrease by a factor of two to one-half its original value.

Considering above both factors

$$I \propto \frac{V}{R}$$

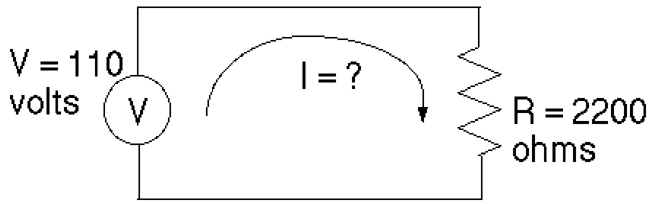
$I = \frac{V}{R}$ (Considering the temperature of the circuit remains constant)

Where I, is the current through the conductor in units of amperes, V is the potential difference measured across the conductor in units of volts, and R is the resistance of the conductor in units of ohms.

$$V = IR$$

$$R = \frac{V}{I}$$

**EXAMPLE – 1.6.1:** A 110-volt wall outlet supplies power to a TV set with a resistance of 2200 ohms. How much current is flowing through the TV?




**Solution:** V = 110 V R = 2200 Ω I =? A

$$I = \frac{V}{R}$$

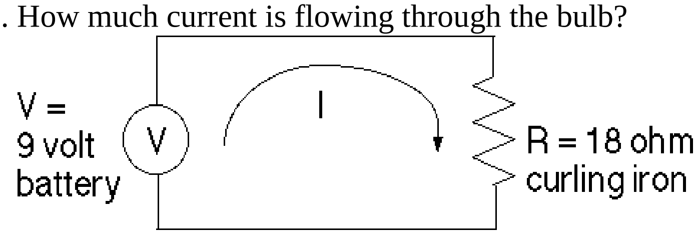
$$= \frac{110}{2200} = 0.05 \text{ A}$$

**EXAMPLE – 1.6.2:** A CD player with a resistance of 2400 ohms has a current of 0.1 amps flowing through it. Calculate how many volts supply is connected with the CD player.

**Solution:** R = 2400 Ω I = 0.1 A V =? V

V = I X R = 0.1 X 2400 = 240 V

**EXAMPLE – 1.6.3:** A nine-volt battery supplies power to a bulb with a resistance of 18 ohms. How much current is flowing through the bulb?




**Solution:**

V = 9 V R = 18 Ω I =? A

$$I = \frac{V}{R} = \frac{9}{18} = 0.5 \text{ A}$$

**EXAMPLE – 1.6.4:** An electric iron is connected with 220 V supply, taking a current of 2.2 A. Find its hot resistance.

**Solution:**

V = 220 V I = 2.2 A R =? Ω

$$R = \frac{V}{I} = \frac{220}{2.2} = 100\ \Omega$$

## 1.7 LAWS OF RESISTANCE:

The resistance of a conductor depends upon four factors;

**1- Length of the material.**

The resistance of a material is directly proportional to the length. The resistance of longer wire is more. Length is denoted by L

$R \propto L$

**2- Cross-sectional area**.

The resistance of a material is inversely proportional to the cross-sectional area of the material. More cross-sectional area allowed the passage of a large number of free electrons, offering less resistance. The cross-sectional area is denoted by 'A'.

$R \propto 1 / A$

**3- The type and nature of the material**:

As discussed earlier whether it consists large number of free electrons or not, affects the value of the resistance. So, material which is conductor has less resistance while an insulator has very high resistance. The effect of nature of material is considered through the constant of proportionality denoted by p (rho) called resistivity or specific resistance of the material.

$R \propto \rho$

**4- Temperature**:

The temperature of the material affects the value of the resistance. Generally, the resistance of the material increases as its temperature increases. Generally, effect of small changes in temperature on the resistance is not considered as it is negligibly small.

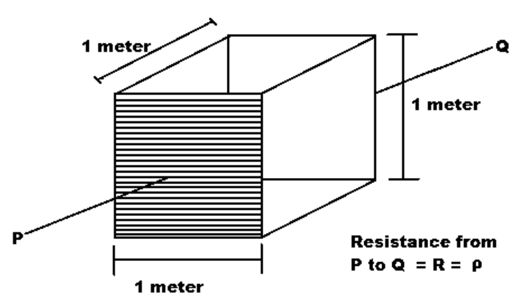
So, for a certain material at a certain temperature we can write a mathematical expression as,

$$R = \frac{\rho L}{A}$$

Where L = length in meters
A = cross-sectional area in square meters
p = resistivity in ohms-meters
R = resistance in ohms

## SPECIFIC RESISTANCE:

Specific resistance, or resistivity, is the resistance in ohms offered by a unit volume (centimeter cube) of a substance to the flow of electric current. The electrical resistivity of a material is denoted by ρ (rho) and is measured in ohm-meters.



The specific resistance of International standard copper is $1.72 \times 10^{-8}$.

## EXAMPLE 1.7.1:

If the resistance of 800m of certain wire is 32ohm. What would be resistance of 250m long?

## Solution:

$$l_1 = 800\,m \qquad l_2 = 250\,m$$

$$R_1 = 32\,\Omega \qquad R_2 = ?$$

$$\frac{R_2}{R_1} = \frac{l_2}{l_1}$$

$$R_2 = \frac{l_2}{l_1} \times R_1 = \frac{250}{800} \times 32 = 10\,\Omega$$

## EXAMPLE 1.7.2:

The resistance of 500m of a certain wire is 125 ohms. What length of the same wire will have a resistance of 90 ohm?

## Solution:

$$R_1 = 125\Omega \qquad R_2 = 90\,\Omega$$

$$l_1 = 500\,m \qquad l_2 = ?$$

$$l_2 = \frac{R_2}{R_1} \times l_1 = \frac{90}{125} \times 500 = 360\,m$$

## EXAMPLE 1.7.3:

Find the resistance of 36.6 meter of copper wire, 0.077 square cm in cross-section and specific resistance of copper 1.7 micro-ohm cm.

## Solution:

$l$ = 36.6 m = 36.6 x 100 cm

a = 0.077 $cm^2$

ρ = 1.7 μ **Ω** - cm

R = ?

$$R = \frac{\rho\, l}{A} = \frac{1.7 \times 10^{-6} \times 36.6 \times 100}{0.077} = 0.08\,\Omega$$

## EXAMPLE 1.7.4:

Find the resistance of 45.7 m iron wire 0.051 cm diameter. Specific resistance of iron is 9.15 μ Ω - cm?

## Solution:

$\rho = 9.15 \times 10^{-6}\ \Omega$ - cm

$l$ = 45.7 m = 45.7 x 100 = 4570 cm

d = 0.051 cm R = ?

$$A = \frac{\pi d^2}{4} = \frac{3.14 \times (0.051)^2}{4} = 0.002\,cm^2$$

$$R = \frac{\rho l}{A} = \frac{9.15 \times 10^{-6} \times 4570}{0.002} = 20.91\Omega$$

## EXAMPLE 1.7.5:

What is the resistance of a copper bar 9.15 m long and 7.62 cm by 1.27 cm in cross – section? Specific resistance of copper is 1.78 μ **Ω** - cm.

## Solution:

$l$ = 9.15 m = 9.15 x 100 =915 cm.

A = Width x Thickness = 7.62 x 1.27 $cm^2$

$\rho$ = 1.78 μ Ω – cm = $1.78 \times 10^{-6}\ \Omega$ - cm

R = ?

$$R = \frac{\rho l}{A} = \frac{1.78 \times 10^{-6} \times 915}{7.62 \times 1.27} = 0.0001683\Omega$$

## EXAMPLE 1.7.6:

If the specific resistance of manganin is 41.8 μ Ω- cm What is the diameter of a manganin wire 68.7-meter-long, which has a resistance of 8.86 Ω.

## Solution:

ρ = 41.8 μ Ω - cm R= 8.86 Ω

$l$ = 68.7m = 68.7 x 100 = 6870 cm

d = ?

$$a = \frac{\rho l}{R} = \frac{41.8 \times 10^{-6} \times 6870}{8.86} = 0.0324\,cm^2$$

$$a = \frac{\pi d^2}{4}$$

$$d^2 = \frac{4a}{\pi} = \frac{4 \times 0.0324}{3.1416} = 0.0412676\,cm^2$$

$$d = \sqrt{0.0412676} \qquad = 0.203\,cm$$

## EXAMPLE 1.7.7:

A wire has resistance of 5.7 ohm. What will be the resistance of another wire of same material three times as long and one half the cross-sectional area?

## Solution:

**In first case:** $R_1 = 5.7\Omega$

$$R_1 = \frac{\rho l_1}{A_1} \qquad \text{or} \qquad \frac{\rho l_1}{A_1} = 5.7\Omega \;\ldots\ldots\ldots\ldots \text{I}$$

**In second case:**

As material is same so $\rho = \rho_1 = \rho_2$

$$l_2 = 3l_1 \qquad A_2 = \frac{1}{2} A_1 \;\; \text{or} \;\; \frac{A_1}{2}$$

$$R_2 = \frac{\rho l_2}{A_2} = \frac{\rho .\, 3l_1}{a_1/2} = \frac{\rho\, 3l_1 \times 2}{a_1} = 6\left(\frac{\rho l_1}{A_1}\right)$$

$$R_2 = 6 \times 5.7 \qquad \text{(From Eq. 1)}$$

$$= 34.2\Omega$$

# Effects of Temperature on Resistance

The resistance of a material changes as the temperature changes. Due to change in temperature, the changes in resistance of different materials are as under.

**1. <u>Pure metals:</u>**

Resistance of pure metals increases as the temperature increases. This increase is excessive and regular for normal limits of temperature. If we draw a graph for resistance and temperature, it is a straight line.

If the resistance of a conductor increases with increase of temperature then the temperature co-efficient of conductor is positive.

**2. Alloys:**

Due to increase in temperature a very small and irregular increase in resistance of alloy. The resistance of alloys used for electrical purposes is practically nearly the same at every temperature.

**3. Non-Metals:**

For non-metals e.g. insulators, Carbon and electrolyte the resistance decreases with increase in temperature which is harmful for an insulator. Their temperature co-efficient is negative.

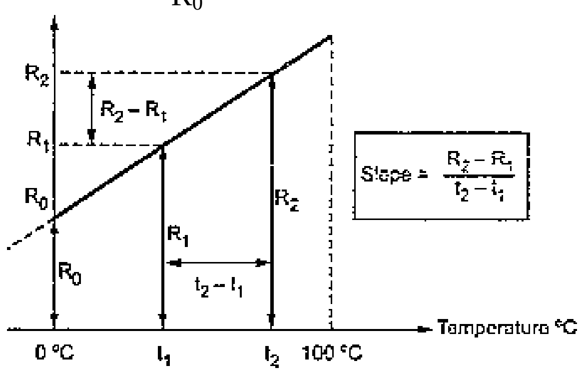
**4. Thermistor:**

Oxides of manganese, copper and nickel are called thermistors. Due to increase in temperature their resistance rapidly decreases, whereas at low temperature their resistance is comparatively greater.

## 1.9 TEMPERATURE CO-EFFICIENT OF A RESISTANCE:

Suppose a resistor has a value $R_0$ at $0^o$ C, then at $1^o$ C, it will increase by a small amount "X", the ratio between X and $R_0$ is called the temperature coefficient of the metal / conductor at $0^o$ C per $^o$C. It is denoted by the symbol $\alpha_0$.

So, $\alpha_0 = \dfrac{X}{R_0}$




***Graph of resistance against temperature***

Resistance at 0° C = $R_0$

Increase in resistance for 1 °C rise of temperature = X

Increase in resistance for $t_1$ °C rise of temperature $= Xt_1$

$R_1$ or $R_t$ = Resistance at $t_1$ °C = Resistance at 0° C + Increase in resistance

$$= R_0 + Xt_1, \quad (X = \alpha_0 R_0)$$
$$= R_0 + \alpha_0 R_0 t_1$$
$$= R_0 (1 + \alpha_0 t_1) \qquad \text{(i)}$$

Where $\alpha_0$ is called temperature coefficient of resistance at 0° C and value of this we can find from equation (i);

$$R_1 \text{ or } R_t = R_0\,(1+\alpha_0 t_1)$$
$$= R_0 + \alpha_0 R_0 t_1$$
$$\alpha_0 R_0 t_1 = R_1 - R_0$$
$$\alpha_0 = \frac{R_1 - R_0}{R_0 t_1}$$

Similarly, for another $t_2$ ° C temperature

$$R_2 = R_0\,(1+\alpha_0 t_2)$$

Sometimes the resistance per degree rise in temperature is referred to the resistance at a temperature other than $0^o$ C, say $t_1$ $^o$C. In this case;

$$\alpha_{t_1} = \frac{X}{R_1}$$
$$X = \alpha_{t_1} R_1$$

Resistance at initial temperature $t_1$ ° C = $R_1$

Resistance at final temperature $t_2$ ° C = $R_2$

Increase in resistance for 1° C rise of temperature = X

Increase in resistance from $t_1$ to $t_2$ ºC rise of temperature = X $(t_1 - t_2)$

Resistance at $t_2$ ° C = Resistance at $t_1$ ºC + x $(t_1 - t_2)$

$$R_2 = R_1 + x\,(t_2 - t_1)$$
$$= R_1 + \alpha_{t_1} R_1\,(t_2 - t_1)$$
$$= R_1 1\,\{1 + \alpha_{t_1}\,(t_2 - t_1)\} \quad \text{(ii)}$$

## EXAMPLE – 1.9.1:

Certain winding made up of copper has a resistance of 100Ωat room temperature. If resistance temperature coefficient of copper at 0 °C is 0.00428 / °C, Find the resistance of winding if temperature is increased to 50 °C. Assume room temperature as 25 °C.

**Solution:**

$t_1$ = 25 °C, $R_1$ = 100 Ω, $t_2$ = 50 °C, $\alpha_0$ = 0.00428 / °C, $R_2$ =? Ω

$$\frac{R_2}{R_1} = \frac{1 + a_0}{1 + a_0}, \frac{t_2 R_2}{t_1 100} = \frac{1 + 0.00428 \times 50}{1 + 0.00428 \times 25}, R_2 = \frac{1.214}{1.107} \times 100$$

$R_2$ = 109.666 Ω

## EXAMPLE – 1.9.2:

The resistance of a wire increases from 40Ω at 10°C to 48.25Ω at 60 °C. Find the temperature co-efficient of resistance at 0 °C.

**Solution:**

$t_1$ = 10 °C, $R_1$ = 40 Ω, $t_2$ = 60 °C, $R_2$ = 48.25 Ω, $\alpha_0$ =? / °C

$\frac{R_2}{R_1} = \frac{1 + a_0}{1 + a_0}$, $\frac{t_2\ 48.25}{t_1 40} = \frac{1 + \alpha_0 \times 60}{1 + \alpha_0 \times 10}$,

By cross multiplication

| | |
|---|---|
| $48.25(1_+ \alpha_0 \text{ X } 10)$ | $= 40\ (1+ \alpha_0 \times 60)$ |
| $48.25 + 482.5\ \alpha_0$ | $= 40 + 2400\ \alpha_0$ |
| $48.25+40$ | $= 2400\ \alpha_0 - 482.5\ \alpha_0$ |
| $8.25$ | $= 1917.5\ \alpha_0$ |
| $\alpha_0$ | $= 8.25 / 1917.5$ |
| | $= 0.0043$ /°C at 0 °C |

## EXAMPLE – 1.9.3:

The field coils of a motor have a resistance of 120 ohms at 15°C. After a run at full load the resistance increases to 135 ohms. Find the average temperature of the coils. Take the resistance temperature coefficient to be 0.00401 per °C at 15°C. (DAE/1A-2009)

## Solution:

| | | | | | |
|---|---|---|---|---|---|
| $R_1$ | = | 120 Ω | $t_1$ | = | 15°C |
| $R_2$ | = | 135 Ω | $\alpha$ | = | 0.00401°C at 15°C |
| $t_2$ | = | ? | | | |

$R_2 = R_1 \{1 + \alpha\ (t_2 - t_1)\}$

$135 = 120\ \{ 1 + 0.00401\ (t_2 - 15)\ \}$

$\frac{135}{120} = 1 + 0.00401\ (t_2 - 15)$

$1.125 - 1 = 0.00401\ (t_2 - 15)$

$\frac{0.125}{0.00401} = t_2 - 15$

$31.172 = t_2 - 15$

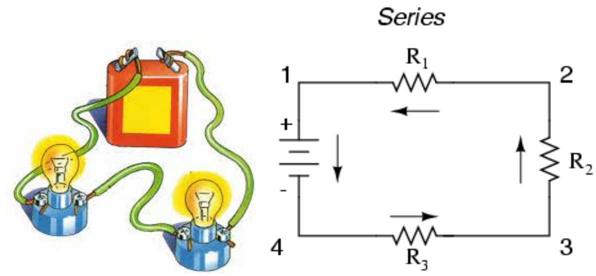
$t_2 = 31.172 + 15 = 46.172$ °C

## 1.10: SERIES & PARALLEL CIRCUIT:

Components of an electrical or electronic circuit can be connected in many different ways. The two simplest of these are called series and parallel and occur very frequently.

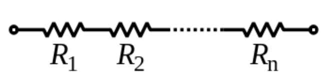
## SERIES CIRCUIT:

Components connected in series are connected along a single path, so the same current flows through all of the components. A circuit composed solely of components connected in series is known as a series circuit. In a series circuit, the current through each of the components is the same; there is only one path in a series circuit in which the current can flow and the voltage across the circuit is the sum of the voltages across each component. As an example, consider a very simple circuit consisting of two 6V light bulbs and one 6 V battery. If a wire joins the battery to one bulb, to the next bulb, then back to the battery, in one continuous loop, the bulbs are said to be in series. If the two light bulbs are connected in series, there is same current through both, and the voltage drop is 3 V across each bulb, which may not be sufficient to make them glow properly. In a series circuit, every device must function for the circuit to be complete. One bulb burning out in a series circuit breaks the circuit.




In the above circuit three resisters are shown connected in series. Series circuits are sometimes called *current*-coupled or daisy chain-coupled. A series circuit's main disadvantage or advantage, depending on its intended role in a product's overall design, is that because there is only one path in which its current can flow, opening or breaking a series circuit at any point causes the entire circuit to "open" or stop operating. For example, if even one of the light bulbs in an older-style string of Christmas tree lights burns out or is removed, the entire string becomes inoperable until the bulb is replaced. Features of series circuit are as follow;

1. **Current:** In a series circuit the current is the same for all elements.
   $I = I_1 = I_2 = I_3 = \ldots\ldots = I_N$
2. **Equivalent resistance:** The total resistance of resistors in series is equal to the sum of their individual resistances:

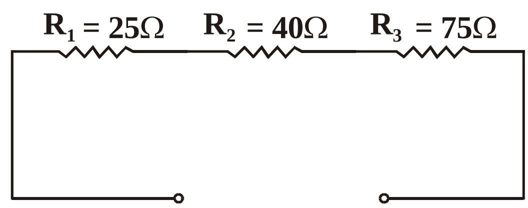



   $R_{total} = R_1 + R_2 + R_3 + \ldots\ldots + R_N$
3. **Voltage:** The total voltage across of resistors in series is equal to the sum of their individual voltage drops.
   $Vt = V1 + V2 + \ldots\ldots + Vn$

## EXAMPLE – 1.10.1:

Three resistors of 25Ω, 40Ω & 75Ω are connected in series. Calculate the equivalent resistance.

**Solution:**




$R_1 = 25\ \Omega$

$R_2 = 40\ \Omega$

$R_3 = 75\ \Omega$

$R_T = ?$

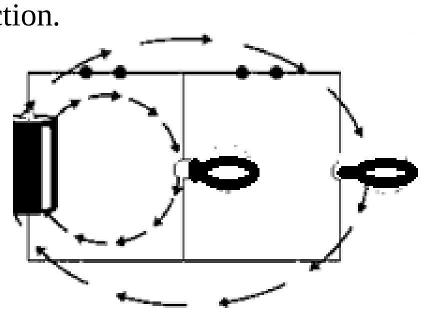
$R_T = R_1 + R_2 + R_3$

$= 25 + 40 + 75 = 140\ \Omega$

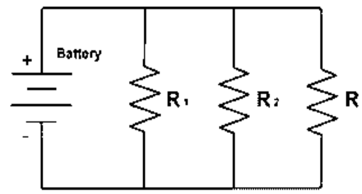
## PARALLEL CIRCUIT:

Components connected in parallel are connected so the same voltage is applied to each component. A circuit composed of components connected completely in parallel is known as a parallel circuit. In a parallel circuit, the voltage across each of the components is the same, and the total current is the sum of the currents through each component.

If each bulb is wired to the battery in a separate loop, the bulbs are said to be in parallel. If the light bulbs are connected in parallel, the currents through the light bulbs combine to form the current in the battery, while the voltage drop is 6.0 V across each bulb and they all glow. In parallel circuits, each light has its own circuit, so all but one light could be burned out, and the last one will still function.



If two or more components are connected in parallel they have the same potential difference (voltage) across their ends. The potential differences across the components are the same in magnitude, and they also have identical polarities. The same voltage is applicable to all circuit components connected in parallel. The total current is the sum of the currents through the individual components.




Features of parallel circuit are as follow**;**

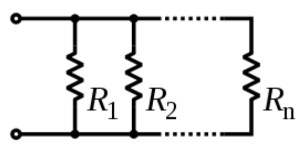
1. **Voltage:** In a parallel circuit the voltage is the same for all elements.

$$V = V_1 = V_2 = V_3 = \ldots\ldots = V_N$$

2. **Current:** The current in each individual resistor is found by Ohm's law. Factoring out the voltage gives

$$\mathbf{I_{total} = I_1 + I_2 + \ldots\ldots.. I_N}$$

3. **Equivalent Resistance:** To find the total resistance of all components, add the reciprocals of the resistances of each component and take the reciprocal of the sum. Total resistance will always be less than the value of the smallest resistance:




$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_N}$$

For only two resistors, the unreciprocated expression is reasonably simple "product over sum".

$$R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

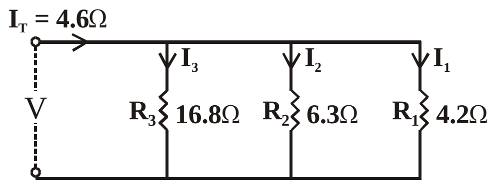
For *N* equal resistors in parallel, the reciprocal sum expression simplifies to:

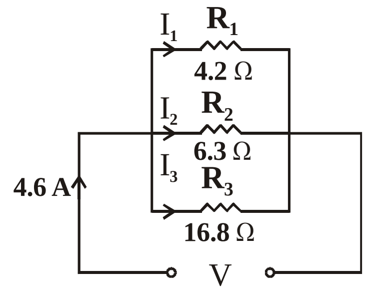
$$\frac{1}{R_{total}} = \frac{1}{R} . N$$

$$R_{total} = \frac{R}{N}$$

## EXAMPLE – 1.11.1:

Three resistors of 4.2, 6.3 and 16.8 ohms are connected in parallel. If the total current taken is 4.6A, find current through each resistor.




(a) Given Diagram (b) Equivalent simple diagram

## Solution:

$R_1 = 4.2\Omega \qquad I_T = 4.6\text{ A}$

$R_2 = 6.3\Omega \qquad I_1, I_2, I_3, = ?$

$R_3 = 16.8\Omega \qquad V = ?$

$$\frac{I}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4.2} + \frac{1}{6.3} + \frac{1}{16.8}$$

$$= 0.238 + 0.158 + 0.06 = 0.456$$

$$R_T = \frac{1}{0.456} = 2.19\Omega$$

$$V_T = I_T R_T = 4.6 \times 2.19 = 10V$$

In parallel circuit $V_T = V_1 = V_2 = V_3 = 10V$

$$I_1 = \frac{V}{R_1} = \frac{10}{4.2} = 2.4A$$

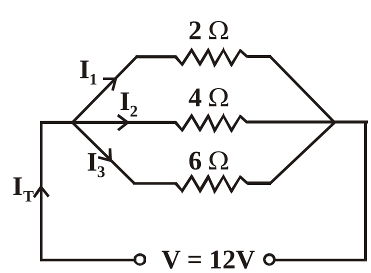
$$I_2 = \frac{V}{R_2} = \frac{10}{6.3} = 1.6A$$

$$I_3 = \frac{V}{R_3} = \frac{10}{16.8} = 0.6A$$

## EXAMPLE – 1.11.2:

A battery with a 12 V, emf and form a parallel circuit with a net work of three resistors 2, 4 & 6 ohm. Calculate.

(a) Current in each resistor
(b) Total current.




**Solution:**

We know that in parallel circuit voltages are same. i.e.,

$$V_T = V_1 = V_2 = V_3 = 12V$$

$$I_1 = \frac{V_1}{R_1} = \frac{12}{2} = 6A$$

$$I_2 = \frac{V_2}{R_2} = \frac{12}{4} = 3A$$

$$I_3 = \frac{V_3}{R_3} = \frac{12}{6} = 2A$$

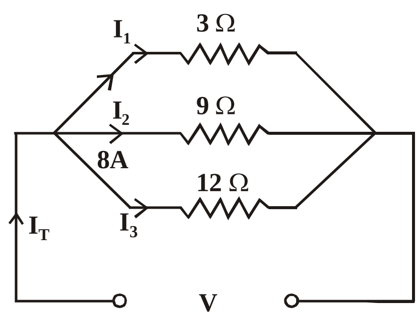
Total Current $= I_T = I_1 + I_2 + I_3$

$$I_T = 6 + 3 + 2 = 11A$$

## EXAMPLE – 1.11.3:

Three resistors of 3, 9 and 12 ohms are joined in parallel and connected to a supply. If the current through the 9ohm resistor is 8A, find the current through the others and the total current taken from the supply.

### Solution:




$R_1 = 3\Omega$ $\quad$ $I_2 = 8A$

$R_2 = 9\Omega$ $\quad$ $I_1, I_3 = ?$

$R_3 = 12\Omega$ $\quad$ $I_T = ?$

$V_2 = I_2R_2 = 8 \times 9 = 72\ V$

In parallel circuit

$V_T = V_1 = V_2 = V_3 = 72V$

$$I_1 = \frac{V_1}{R_1} = \frac{72}{3} = 24A$$

$$I_3 = \frac{V_3}{R_3} = \frac{72}{12} = 6A$$

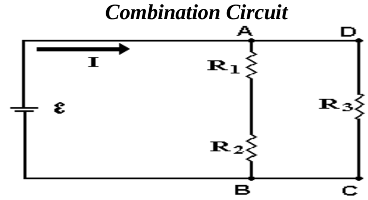
Total Current $= I_T = I_1 + I_2 + I_3$

$$I_T = 24 + 8 + 6 = 38A$$

## 1.12 RESISTORS IN COMBINATION CIRCUITS:

Now, we can replace the two resistors with a single, equivalent resistor with no effective change to the circuit.

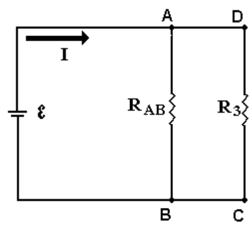
*Combination Circuit*



Here, we will combine series circuits and parallel circuits. These are known as combination circuits. No new equations will be learned here. We can imagine a branch in a parallel circuit, but which contains two resistors in series. For example, between points A and B in Figure.

In this situation, we could calculate the equivalent resistance of branch AB using our rules for series circuits. So,

$$R_{AB} = R_1 + R_2$$



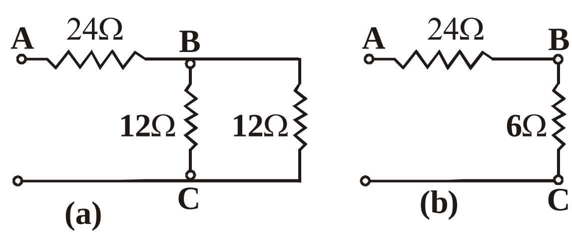

***simplified parallel circuit***

As can be seen in Figure, the circuit is now a parallel circuit, with resistors $R_{AB}$ and $R_3$ in parallel. This circuit can be solved using the same rules as any other parallel circuit.

## EXAMPLE – 1.12.1:

Find the equivalent resistance of the circuit shown in parallel:

**Solution:**




$$R_{BC} = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{12 \times 12}{12 + 12}$$

$$= \frac{144}{24} = 6\Omega$$

Now it is a series circuit as shown in diagram (b)

$$R_{ABC} = R_{AB} + R_{BC}$$

$$= 24 + 6 = 30\Omega$$

$R_{ABC} = R_T = 30\Omega$ Ans.

## 1.13 DRAW EQUIVALENT CIRCUIT OF COMPLEX NETWORK:

In electrical engineering and science, an equivalent circuit refers to a theoretical circuit that retains all of the electrical characteristics of a given circuit. Often, an equivalent circuit is sought that simplifies calculation, and more broadly, that is a simplest form of a more complex circuit in order to aid analysis. In its most common form, an equivalent circuit is made up of linear, passive elements. However, more complex equivalent circuits are used that approximate the nonlinear behavior of the original circuit as well. These more complex circuits often are called macro models of the original circuit. An example of a macro model is the Boyle circuit for the 741 operational amplifiers.

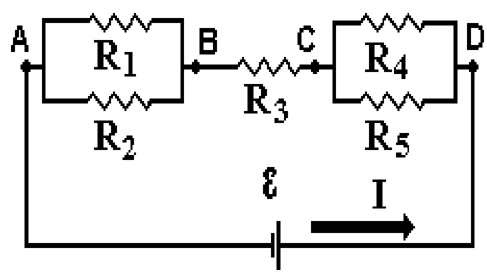
Equivalent circuits can also be used to electrically describe and model either;

a) Continuous materials or biological systems in which current does not actually flow in defined circuits.

b) Distributed reactance's such as found in electrical lines or windings that do not represent actual discrete components. For example, a cell membrane can be modeled as a capacitance (i.e. the lipid bilayer) in parallel with resistance-DC voltage source combinations (i.e. ion channels powered by an ion gradient across the membrane).

For illustration here, the resistors in the parallel circuit AB can be replaced by an equivalent resistance. Again, we will use the equivalence rule for resistors connected in parallel:

$$\frac{1}{R_{equivalent}} = \Sigma \frac{1}{R_i}$$

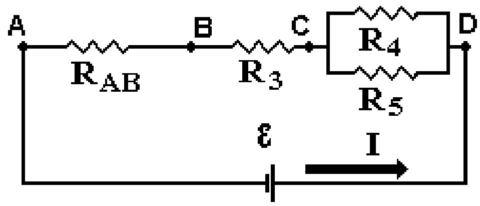


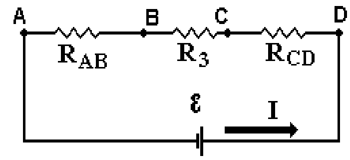

*Combination Circuit*

This gives:

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2}$$

So, the equivalent resistance between points A and B is RAB. Replacing the parallel circuit between these two points with RAB gives the following circuit.




*simplified circuit*

*partially simplified circuit*

Similarly, we can replace the parallel circuit containing R4 and R5 (between points C and D) with its equivalent resistance, RCD, where

$$\frac{1}{R_{CD}} = \frac{1}{R_4} + \frac{1}{R_5}$$

Replacing the parallel circuit between CD with its equivalent resistance yields the circuit in above Figure. Now, you can see that we have simplified Circuit to one which contains resistors connected in series only. That is, this circuit now contains RAB, R3, and RCD in series. The equivalent resistance for this circuit would be found using:

$$R_{equivalent} = \sum R_i$$

or

$$R_{Total} = RAB + R3 + RCD$$

Here is an interesting animated exercise to help you with these concepts and Ohm's Law

At each step, write down the given values such as voltage, current, resistance.

If solving Series-Parallel circuits solve the Parallel parts first, and then you are left with only a much-easier Series circuit.

In parallel circuits and series-parallel, you will often find "t" added to any of these symbols, in which case it represents Total, meaning the voltage, current, or resistance of the circuit when considered as a whole.

The sum of the individual currents will equal the total current It=I1+I2+I3...

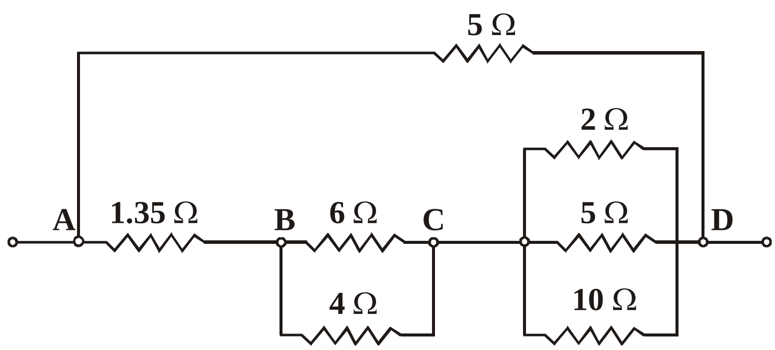
The sum of the individual Powers will equal the total power Pt=P1+P2...

In a Parallel circuit the same voltage is applied across all the resistors.

## EXAMPLE: 1.13.1:

Find the equivalent resistance of the circuit shown:

## Solution:



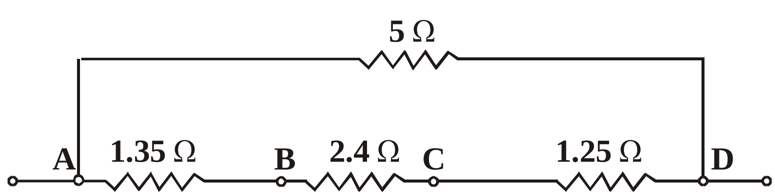

$$R_{BC} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 4}{6 + 4} = \frac{24}{10} = 2.4\Omega$$

$$\frac{1}{R_{CD}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2} + \frac{1}{5} + \frac{1}{10} = \frac{5 + 2 + 1}{10} = \frac{8}{10}$$

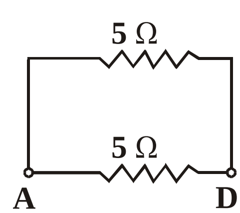
$$R_{CD} = \frac{10}{8} = 1.25\ \Omega$$

Now the circuit is like as:




$$R_{ABCD} = R_{AB} + R_{BC} + R_{CD}$$

$$= 1.35 + 2.4 + 1.25 = 5\Omega$$




$$R_T = R_{AD} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{5 \times 5}{5 + 5} = \frac{25}{10} = 2.5\ \Omega$$

# EXERCISE # 01

## PART-A
## MULTIPLE CHOICE QUESTIONS

**Encircle the correct answer.**

1. The particles in the nucleus of an atom are:
   (a) Electron and Proton (b) Electron and Neutron
   (c) Proton and Neutron (d) Only Neutron
2. Negative Charge on:
   (a) Electrons (b) Protons (c) Neutrons (d) Both b & c
3. Charge on Neutrons is:
   (a) Positive (b) Negative (c) Any one (d) Not any
4. The maximum numbers of electrons on outermost shell is:
   (a) 2 (b) 8 (c) 18 (d) 32
5. Numbers of electrons in outer most shell of copper atom is:
   (a) 1 (b) 2 (c) 3 (d) 4
6. Outer most shell of atom is called:
   (a) External (b) Internal (c) Valance (d) Both b & c
7. Number of electrons in outer most shell of conducting material is:
   (a) More than 4 (b) Always 4 (c) Less than 4 (d) 8
8. Number of electrons in outer most shell of insulating material is:
   (a) More than 4 (b) Always 4 (c) Less than 4 (d) 2
9. Number of electrons in outer most shell of semi-conductor material is:
   (a) More than 4 (b) Always 4 (c) Less than 4 (d) 2
10. Deficiency or excess of electrons of atom is called:
    (a) Voltage (b) Current (c) Charge (d) Potential
11. The SI unit of electrical resistance is:
    (a) Ampere (b) Volt (c) Ohm (d) Watt
12. The inverse quantity of resistance is called:
    (a) Voltage (b) Power (c) Impedance (d) Conductance
13. The property of material or a conductor, which can pass current very easily is called:
    (a) Resistance (b) Conductance (c) Impedance (d) None
14. Charge of 6.24 × 1018 electrons is:
    (a) One coulomb (b) One ampere (c) One watt (d) One meter
15. Rate of flowing of free electrons in any conductor is called:
    (a) Volt (b) Ohm (c) Watt (d) Current
16. The current which magnitude and direction not changed, is called:
    (a) AC (b) DC (c) Pulsating DC (d) Both a & b
17. The current which magnitude and direction change continuously:

(a) DC (b) AC (c) Pulsating DC (d)Both a & b

18. Unit of current is:
(a) Volt (b) Ampere (c) Watt (d) Ohm

19. According to Ohm's law:
(a) V = IR (b) V = I/R (c) V = R/I (d) All of these

20. Formula to determine of current is:
(a) VR (b) V/R (c) R/V (d) V/W

21. The resistance of conductor material is directly proportional to the:
(a) Width (b) Length (c) Area (d) Volume

22. The best conducting material is:
(a) Silver (b) Copper (c) Aluminium (d) Iron

23. The resistance of pure metals --------- with increasing of temperature:
(a) Increases (b) Decreases
(c) Remains constant (d) Becomes zero

24. The number of paths for the flow of current; in series circuit:
(a) Many (b) Two (c) Four (d) One

25. In parallel circuits:
(a) $I = I_1 = I_2 = I_3 .... = I_n$ (b) $I = I_1 - I_2 - I_3 .... - I_n$
(c) $I = I_1 + I_2 + I_3 .... + I_n$ (d) All of these

26. Temperature co-efficient of resistance is denoted by:
(a) b (b) r (c) µ (d) W

27. Specific resistance is denoted by:
(a) b (b) r (c) µ (d) W

28. Two resistors of 4 and 6 ohms are connected in series, the total resistance will be:
(a) 10 ohms (b) 6 ohms (c) 4 ohms (d) 24 ohms

29. By increasing the resistors in parallel circuit total resistance:
(a) Increases (b) Decreases
(c) Remains constant (d) Highly increases

30. Two resistance $R_1$ and $R_2$ are connected in parallel, the formula for the total resistance is:
(a) $R_T = R_1 + R_2/R_1R_2$ (b) $R_T = R_1R_2/R_1 + R_2$
(c) $R_T = R_1 + R_2$ (d) $R_T = R_1R_2$

## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | a | 3. | d | 4. | b | 5. | a |
| 6. | c | 7. | c | 8. | a | 9. | b | 10. | c |
| 11. | c | 12. | d | 13. | b | 14. | a | 15. | d |
| 16. | b | 17. | b | 18. | b | 19. | a | 20. | b |
| 21. | b | 22. | a | 23. | a | 24. | b | 25. | c |
| 26. | c | 27. | b | 28. | a | 29. | b | 30. | b |

# PART-B
# SHORT QUESTIONS WITH ANSWERS

**Write short answers of the following questions.**

1. Define the atom.
2. Write the fundamental parts of atom.
3. Draw the atomic structure of Boron.
4. Define conductor.
5. Define Insulator.
6. Define Semi-Conductor.
7. Define the resistance & write its unit.
8. Define conductance.
9. Define Current & write its unit.
10. Define Ampere.
11. State Ohm's law.
12. An electric iron is connected with 200V supply, taking a current of 2A. Find its hot resistance.
13. Enlist laws of resistance.
14. Define specific resistance.
15. Define temperature co-efficient of resistance.
16. Ten resistances are connected in parallel, each resistance of 1000 ohm. Find the total resistance.
17. Define Series Circuit.
18. State 4 properties of Series Circuit.
19. Define Parallel Circuit.
20. State 4 properties of Parallel Circuit.
21. Write the formula to calculate three resistance connected in Series.
22. Write the formula to calculate three resistance connected in parallel.
23. Two resistors of 10W and 15W are connected in parallel, calculate total resistance. Write its formula too.

# PART-C
# LONG QUESTIONS

1) Explain electron theory.
2) Compare conductor, semi-conductor & Insulator & give examples.
3) Explain Ohm's Law.
4) Describe specific resistance and write its formula.
5) Describe laws of resistance.
6) Describe the effects of temperature on resistance.
7) Explain series circuit with the help of diagram and state its characteristics.

8) Explain Parallel circuit with the help diagram and state its characteristics.

9) Compare Series Circuit and a Parallel Circuit.

## PART-D

**Q.1 The hot resistance of 110volt carbon filament lamp is 240 ohms. Find the current:**

**Ans.** I = 0.458A

**Q.2 An electric iron takes 2.3 ampere at 230 volts. What is its hot resistance?**

**Ans.** R = 100 Ω

**Q.3 How much voltage will be measured across the resistor from 4mA current through 60 KΩ resistances.**

**Ans.** V = 240 V

**Q.4 A coil has a current of 50 mA flowing through it when the applied voltage is 12 volts. What is the resistance of the coil?**

**Ans.** R = 240 Ω

**Q.5 What is the resistance of a coil which draws a current of 50mA from a 120 volts supply?**

**Ans.** R = 2.4 Ω

**Q.6 Determine the voltage which must be applied to a 2kΩ resistor in order that a current of 10mA may flow.**

**Ans.** V = 20 V

**Q.7 The hot resistance of 240 volts filament lamp is 960Ω. Find the current taken by the lamp?**

**Ans.** I = 0.25A

**Q.8 An electromagnet of resistance 12.4 ohm requires a current of 1.5 ampere to operate it. Find the required voltage?**

**Ans.** V = 18.6 V

**Q.9 The cold resistance of a certain gas-filled tungsten lamp is 18.2 ohms and its hot resistance at the operating voltage of 110 volts is 202 ohms. Find the current: -**

**Ans.** I = 0.544A

**Q.10 The resistance of a conductor 1mm$^2$ cross-section and 20m long, is 0.346Ω. Determine the specific resistance of the conductor.**

**Ans.** ρ = 1.73 × 10–8Ωm

**Q.11** **A coil consists of 2,000 turns of copper wire having a cross-sectional area of 0.8 $mm^2$. The mean length per turn is 80cm and the resistivity of copper is $2 \times 10^{-8}\Omega$m. Find the resistance.**

**Ans.** $R = 40\Omega$

**Q.12** **Find the length of a piece of Nichrome wire 0.102cm in diameter, which has a resistance of 25Ω. Specific resistance of Nichrome is 109μ Ω– cm**

**Ans.** $\ell$ = 18.738m

**Q.13** **A column of mercury has a resistance of 10 ohms at 15°C. What will be its resistance at 30°C? The resistance-temperature co-efficient of mercury is 0.0072 per°C at 0°C.**

**Ans.** $\mathbf{R_2 = 10.974\Omega}$

**Q.14** **A copper coil has a resistance of 0.4 ohm at 12°C. Find its resistance at 52°C. Resistance-temperature co-efficient 0.004 per °C at 0°C.**

**Ans.** $R_2 = 0.461\Omega$

**Q.15** **The resistance-temperature co-efficient of phosphor bronze is 0.00394 per °C at 0°C.A length of phosphor bronze wire has a resistance of 10ohms at 20°C. What is its resistance at 60°C?**

**Ans.** $\mathbf{R_2 = 11.46\Omega}$

**Q.16** **A coil has a resistance of 20 ohms at 25°C. Find its resistance at 65°C. Resistance-temperature co-efficient at 25°C = 0.00385 per °C.**

**Ans.** $\mathbf{R_2 = 23.08\Omega}$

**Q.17** **The field coils of a motor have a resistance of 120ohms at 15°C. After a run at full load the resistance increases to 135 ohms. Find the average temperature of the coils. Take the resistance temperature co-efficient to be 0.00401 per °C at 15°C.**

**Ans.** $t_2 = 46.172°C$

**Q.18** **A resistor of 20Ω is connected in parallel with 5Ω resistor. This combination is now joined in series with 8Ω resistor. What will be the total resistance of circuit?**

**Ans.** $R_T = 12\Omega$

**Q.19** **Two resistors of 20Ω & 40Ω are connected in series. now this combination is jointed in parallel with 30Ω resistor. Find the equivalent resistance of the circuit.**

**Ans.** $\mathbf{R_T = 20\Omega}$

**Q.20 Three resistors of 10 Ω, 20 Ω & 30 Ω are connected in series across a 120-volt supply. Calculate:**

**(i) Total resistance (ii) Current**

**(iii) Voltage drops across each resistance**

**Ans.** $R_T = 60\ \Omega$ $I_T = 2A$

$V_1 = 20$ V $V_2 = 40$ V $V_3 = 60$ V

**Q.21 Three resistors of 8.4, 6.8 and 4.8 ohms are connected in series across a 100-volt supply. Find:**

**(a) Total resistance (b) Current**

**(c) Voltage across each resistor**

**Ans.** $R_T = 20\ \Omega$ $I_T = 5A$

$V_1 = 42$ V $V_2 = 34$ V $V_3 = 24$ V

**Q.22 Two resistors of 4 ohm & 6 ohm are connected in parallel. If the total current taken is 30 amperes, find the current through each.**

**Ans. $I_2 = 12$ Ampere**

**Q.23 Two resistors of 3 Ω & 6 Ω are connected in parallel across 18-volt supply. Find:**

**(i) Total resistance (ii) Current in each resistor**

**(iii) Total current**

**Ans.** $V_T = 18$ V $I_2 = 3A$ $I_T = 9A$

**Q.24 Three resistors of 2, 5 and 10 ohms are joined in parallel and total current of 24 A is passed through them. Find the current through each.**

**Ans. $I_3 = 3$ A**

# Chapter # 2

# KIRCHHOFF'S LAWS

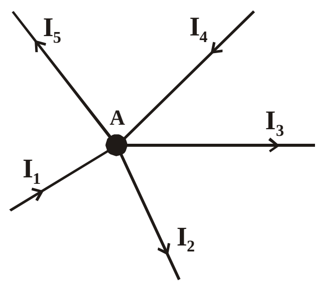
## 2.1- KIRCHHOFF'S 1ST LAW:

This law is also called Kirchhoff's current law, Kirchhoff's point rule, or Kirchhoff's junction rule (or nodal rule).

The principle of conservation of electric charge implies that:

At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node, or:

The algebraic sum of currents in a network of conductors meeting at a point is zero.




$$\Sigma I = 0$$

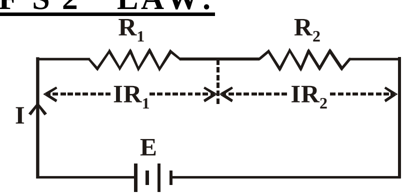
Recalling that current is a signed (positive or negative) quantity reflecting direction towards or away from a node; this principle can be stated as:

Sum of positive currents = Sum of negative currents

**USES**

A matrix version of Kirchhoff's current law is the basis of most circuit simulation software, such as SPICE. Kirchhoff's current law combined with Ohm's Law is used in nodal analysis.

## 2.2: KIRCHHOFF'S 2ND LAW:




The sum of all the voltages around the loop is equal to zero.

$$V_1 + V_2 + V_3 - E = 0$$

This law is also called Kirchhoff's Voltage law, Kirchhoff's loop (or mesh) rule, and Kirchhoff's second rule.

The principle of conservation of energy implies that;
The directed sum of the electrical potential differences (voltage) around any closed network is zero, or:
More simply, the sum of the emf in any closed loop is equivalent to the sum of the potential drops in that loop, or:

The algebraic sum of the products of the resistances of the conductors and the currents in them in a closed loop is equal to the total emf available in that loop.

It can be stated as:

$$\Sigma\, IR = \Sigma\, emf$$

## 2.3.1 ACTIVE CIRCUIT:

It is a circuit which contains one or more than one sources of EMF in it.

## 2.3.2 PASSIVE CIRCUIT:

It is a circuit which contains no supply source of EMF in it.

## 2.3.3 BRANCH:

That part of a network, which lies between two junctions, is called a branch. In a branch there can be one or more elements in series.

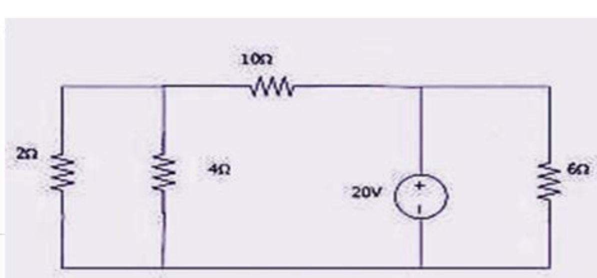
## 2.3.4 NODE:

Node refers to any point on a circuit where two or more circuit elements meet. For two nodes to be different, their voltages must be different. Without any further knowledge, it is easy to establish how to find a node by using Ohm's Law: V=IR. When looking at circuit schematics, ideal wires have a resistance of zero. Since it can be assumed that there is no change in the potential across any part of the wire, all of the wire in between any components in a circuit is considered part of the same node.
Voltage = Current x Resistance

Since voltage is a measure of potential difference the voltage between any two parts of the same node is: $V_{ab}$= (Current) x 0
So, at any two points on the same branch of the circuit, the change in potential difference is 0. Therefore, throughout the entire node the voltage is the same.

## EXAMPLE:

Now tell how many branches and nodes are in below circuit?



**Answer:**

1- The circuit has five branches as it contains five elements.

2- It has three nodes, because short path is counted one node.

## 2.3.5 MESH:

It is a loop or closed path, where there is only one path for passing of current.

## 2.3.6 LOOP:

A loop is any closed path in a circuit. Loop counts starting at a node passing through a set of nodes and returning to the starting node without passing through any node more than once. A loop is said to be independent if it contains at least one branch which is not a part of any other independent loop. From independent path or independent loop, we get independent set of the equations. We can consider above circuit to define set of loops. For 4Ω resistor consider with independent 20V voltage source their path abca is a loop. If we think second loop for 6Ω with independent 4A current source then we get another loop. Such way many set of loops can be made.

## 2.4 APPLICATIONS OF KIRCHHOFF'S LAW:

Kirchhoff's rules can be used to analyze any circuit and modified for those with EMFs, resistors, capacitors and more. Practically speaking, however, the rules are only useful for characterizing those circuits that cannot be simplified by combining elements in series and parallel.

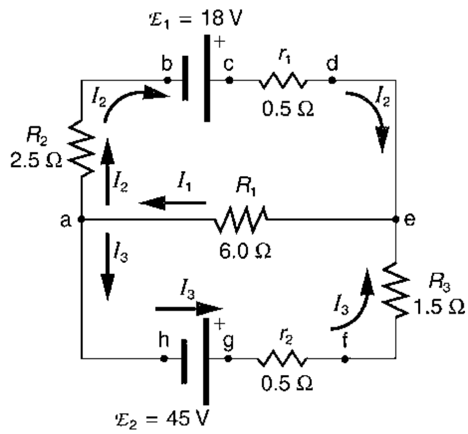
Combinations in series and parallel are typically much easier to perform than applying either of Kirchhoff's rules, but Kirchhoff's rules are more broadly applicable and should be used to solve problems involving complex circuits that cannot be simplified by combining circuit elements in series or parallel.

## KEY POINTS:

- Kirchhoff's rules can be applied to any circuit, regardless of its composition and structure.
- Because combining elements is often easy in parallel and series, it is not always convenient to apply Kirchhoff's rules.
- To solve for current in a circuit, the loop and junction rules can be applied. Once all currents are related by the junction rule, one can use the loop rule to obtain several equations to use as a system to find each current value in terms of other currents. These can be solved as a system.

## EXAMPLE OF KIRCHHOFF'S RULES – 2.14.1:

Figure shows a very complex circuit, but Kirchhoff's loop and junction rules can be applied. To solve the circuit for currents $I_1$, $I_2$, and $I_3$, both rules are necessary.




Applying Kirchhoff's junction rule at point a, we find:

- Because $I_1$ flows into point a, while $I_2$ and $I_3$ flow out. The same can be found at point e. We now must solve this equation for each of the three unknown variables, which will require three different equations.
- Considering loop abcdea, we can use Kirchhoff's loop rule:
- Substituting values of resistance and emf from the figure diagram and canceling the ampere unit gives:
- This is the second part of a system of three equations that we can use to find all three current values. The last can be found by applying the loop rule to loop aefgha, which gives:
- Using substitution and simplifying, this becomes:
- In this case, the signs were reversed compared with the other loop, because elements are traversed in the opposite direction.
- We now have three equations that can be used in a system. The second will be used to define $I_2$, and can be rearranged to:
- The third equation can be used to define $I_3$, and can be rearranged to:
- Substituting the new definitions of $I_2$ and $I_3$ (which are both in common terms of $I_1$), into the first equation ($I_1=I_2+I_3$), we get:
- Simplifying, we find that $I_1$=4.75A. Inserting this value into the other two equations, we find that $I_2$=-3.50 A and $I_3$=8.25 A.

## Solution:

According to the first law we have

$$i_1 - i_2 - i_3 = 0$$

$$i_3 = i_1 - i_2$$

The second law applied to the closed circuit $s_1$ gives

$R_1 i_1 + R_2 i_2 = \Sigma$ E

$6 i_1 + 3 i_2 = 18$

$6\ i_1 + 3\ i_2 = 18$ -------------- i

The second law applied to the closed circuit $s_2$ gives

$R_3 i_3 + R_2 i_1 = \Sigma$ E

$2\ i_3 - 6 i_1 = 45$

$3(i_1 - i_2) - 6 i_1 = 45$

$8\ i_1 - 2 i_2 = 45$ -------------- ii

Multiplying equation I with 2

$12\ i_1 + 6\ i_2 = 36$ --------------iii

Multiplying equation ii with 3

$24\ i_1 - 6 i_2 = 135$ --------------iv

Adding equation iii &iv

$36\ i_1 = 171$

$$i_1 = \frac{171}{36} = 4.75\text{ A}$$

Putting this value in equation i

$6 \text{ X } 4.75 + 3\ i_2 = 18$

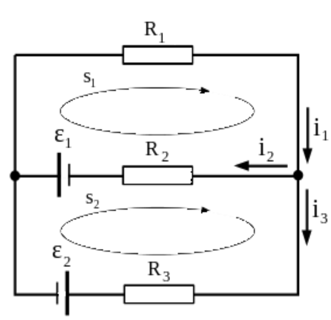
$3\ i_2 = 18 - 28.5$

$$i_2 = \frac{-10.5}{3} = -3.5\text{ A}$$

$$i_3 = i_1 - i_2 = 4.75 + 3.5 = 8.25\text{ A}$$

The negative sign of $i_2$ means that the direction of $i_2$ is opposite to the assumed direction (the direction defined in the picture).

## EXAMPLE – 2.5.2:




Assume an electric network consisting of two voltage sources and three resistors. $R_1 = 100,\ R_2 = 200,\ R_3 = 300$ (ohms); $\epsilon_1 = 3,\ \epsilon_2 = 4$ (volts)

According to the first law we have

$$i_1 - i_2 - i_3 = 0$$

$$i_3 = i_1 - i_2$$

The second law applied to the closed circuit $s_1$ gives

$$-R_2 i_2 + \epsilon_1 - R_1 i_1 = 0$$
$$-200 i_2 + 3 - 100 i_1 = 0$$
$$100\ i_1 + 200\ i_2 = 3 \qquad \text{-------------- i}$$

The second law applied to the closed circuit $s_2$ gives

$$-R_3 i_3 - \epsilon_2 - \epsilon_1 + R_2 i_2 = 0$$
$$-300\ i_3 - 4 - 3 + 200 i_2 = 0$$
$$-300(i_1 - i_2) - 4 - 3 + 200 i_2 = 0$$
$$-300\ i_1 + 500\ i_2 = 7 \text{ -------------- ii}$$

Multiplying equation, i with 3

$$300\ i_1 + 600\ i_2 = 9 \qquad \text{--------------iii}$$

Adding equation ii & iii

$$1100\ i_2 = 16$$
$$i_2 = \frac{16}{1100} = 0.0145 \text{ A}$$

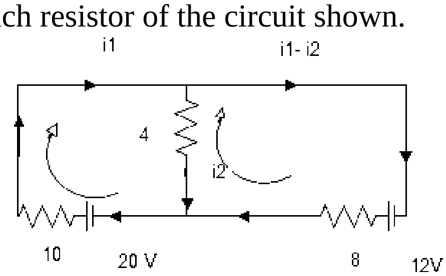
Putting this value in equation i

$$100\ i_1 + 200 \text{ X } 0.0145 = 3$$
$$100\ i_1 = 3 - 2.9$$
$$i_1 = \frac{0.1}{100} = 0.001 \text{ A}$$
$$i_3 = i_1 - i_2 = 0.001 - 0.0145 = -0.0135 \text{ A}$$

The negative sign of $i_3$ means that the direction of $i_3$ is opposite to the assumed direction (the direction defined in the picture).

## EXAMPLE - 2.5.3:

Find the current in each resistor of the circuit shown.




According to the first law we have

$$i_1 - i_2 - i_3 = 0$$
$$i_3 = i_1 - i_2$$

The second law applied to the closed circuit $s_1$ gives

$$R_1 i_1 + R_2 i_2 = \Sigma \text{ E}$$
$$10 i_1 + 4 i_2 = 20$$
$$10 i_1 + 4 i_2 = 20 \qquad \text{-------------- i}$$

The second law applied to the closed circuit $s_2$ gives

$R_3 i_3 + R_2 i_2 = \Sigma$ E

$8 i_3 - 4 i_2 = 12$

$8(i_1 - i_2) - 4 i_2 = 12$

$8 i_1 - 12 i_2 = 12$-------------- ii

Multiplying equation, i with 3

$30\ i_1 + 12 i_2 = 60$ --------------iii

Adding equation ii & iii

$38 i_1 = 72$

$$i_1 = \frac{72}{38} = 1.895 \text{ A}$$

Putting this value in equation i

$10X\ 1.895 + 4\ i_2 = 20$

$4 i_2 = 20 - 18.95$

$$i_2 = \frac{1.05}{4} = 0.263 \text{ A}$$

$$i_3 = i_1 - i_2 = 1.895 - 0.263 = 1.63 \text{ A}$$

## 2.6 SUPER POSITION THEOREM:

The superposition theorem for electrical circuits states that for a linear system the response (voltage or current) in any branch of a bilateral linear circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, where all the other independent sources are replaced by their internal impedances.

To ascertain the contribution of each individual source, all of the other sources first must be "turned off" (set to zero) by:

1. Replacing all other independent voltage sources with a short circuit (thereby eliminating difference of potential i.e. V=0; internal impedance of ideal voltage source is zero (short circuit)).
2. Replacing all other independent current sources with an open circuit (thereby eliminating current i.e. I=0; internal impedance of ideal current source is infinite (open circuit)).

This procedure is followed for each source in turn, and then the resultant responses are added to determine the true operation of the circuit. The resultant circuit operation is the superposition of the various voltage and current sources.

The superposition theorem is very important in circuit analysis. It is used in converting any circuit into its Norton equivalent or Thevenin's equivalent.

The theorem is applicable to linear networks (time varying or time invariant) consisting of independent sources, linear dependent sources, linear passive elements (resistors, inductors, capacitors) and linear transformers.
Another point that should be considered is that superposition only works for voltage and current but not power. In other words, the sum of the powers of each source with the other sources turned off is not the real consumed power. To calculate power, we should first use superposition to find both current and voltage of each linear element and then calculate the sum of the multiplied voltages and currents.

## 2.7 MAXIMUM POWER TRANSFER THEOREM:

In electrical engineering, the maximum power transfer theorem states that, to obtain maximum external power from a source with a finite internal resistance, the resistance of the load must equal the resistance of the source as viewed from its output terminals. Moritz von Jacobi published the maximum power (transfer) theorem around 1840; it is also referred to as "Jacobi's law".

The theorem results in maximum power transfer, and not maximum efficiency. If the resistance of the load is made larger than the resistance of the source, then efficiency is higher, since a higher percentage of the source power is transferred to the load, but the magnitude of the load power is lower since the total circuit resistance goes up.

If the load resistance is smaller than the source resistance, then most of the power ends up being dissipated in the source, and although the total power dissipated is higher, due to a lower total resistance, it turns out that the amount dissipated in the load is reduced.

The theorem states how to choose (so as to maximize power transfer) the load resistance, once the source resistance is given. It is a common misconception to apply the theorem in the opposite scenario. It does *not* say how to choose the source resistance for a given load resistance. In fact, the source resistance that maximizes power transfer is always zero, regardless of the value of the load resistance.

The theorem can be extended to AC circuits that include reactance, and states that maximum power transfer occurs when the load impedance is equal to the complex conjugate of the source impedance.

## 2.8 THEVENIN'S THEOREM:

As originally stated in terms of DC resistive circuits only, the Thevenin's theorem holds that:

- Any linear electrical network with voltage and current sources and only resistances can be replaced at terminals A-B by an equivalent voltage source $V_{th}$ in series connection with an equivalent resistance $R_{th}$.

- This equivalent voltage $V_{th}$ is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
- This equivalent resistance $R_{th}$ is the resistance obtained at terminals A-B of the network with all its independent current sources open circuited and all its independent voltage sources short circuited.

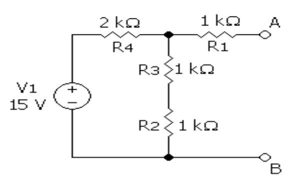
In circuit theory terms, the theorem allows any one-port network to be reduced to a single voltage source and single impedance.

The theorem also applies to frequency domain AC circuits consisting of reactive and resistive impedances. The theorem was independently derived in 1853 by the German scientist Hermann von Helmholtz and in 1883 by Léon Charles Thevenin (1857–1926), an electrical engineer with France's national Posttest telecommunications organization.

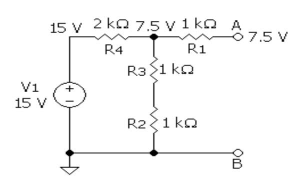
Thevenin's theorem and its dual, Norton's theorem, are widely used for circuit analysis simplification and to study circuit's initial-condition and steady-state response. Thevenin's theorem can be used to convert any circuit's sources and impedances to a Thevenin equivalent; use of the theorem may in some cases be more convenient than use of Kirchhoff's circuit laws.

## EXAMPLE–2.8.1:

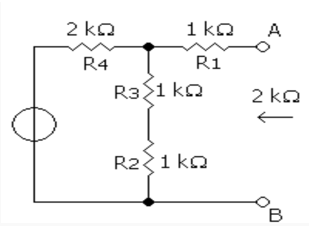
Calculate current in $R_1$ (3 KΩ) connected across point AB by using Thevenin's theorem in the circuit shown.



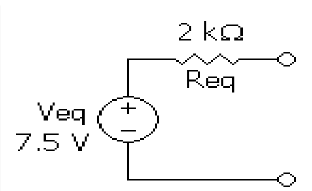

**Step 0: The original Circuit**




**Step 1: Calculating the equivalent output voltage**




**Step 2: Calculating the equivalent resistance**




**Step 3: The equivalent circuit**

In the example, calculating the equivalent voltage:

$$V_{\mathrm{Th}} = \frac{R_2 + R_3}{(R_2 + R_3) + R_4} \cdot V_1$$

$$= \frac{1\,\mathrm{k\Omega} + 1\,\mathrm{k\Omega}}{(1\,\mathrm{k\Omega} + 1\,\mathrm{k\Omega}) + 2\,\mathrm{k\Omega}} \cdot 15\,\mathrm{V}$$

$$= \frac{1}{2} \cdot 15\,\mathrm{V} = 7.5\,\mathrm{V}$$

(notice that $R_1$ is not taken into consideration, as above calculations are done in an open circuit condition between A and B, therefore no current flows through this part, which means there is no current through $R_1$ and therefore no voltage drop along this part)

Calculating equivalent resistance:

$$\begin{aligned} R_{\mathbf{Th}} &= R_1 + [(R_2 + R_3) \| R_4] \\ &= 1\,\mathrm{k\Omega} + [(1\,\mathrm{k\Omega} + 1\,\mathrm{k\Omega}) \| 2\,\mathrm{k\Omega}] \\ &= 1\,\mathrm{k\Omega} + \left( \frac{1}{(1\,\mathrm{k\Omega} + 1\,\mathrm{k\Omega})} + \frac{1}{(2\,\mathrm{k\Omega})} \right)^{-1} = 2\,\mathrm{k\Omega}. \end{aligned}$$

Now $R_t = R_{Th} + R_1 = 2 + 3 = 5\ \mathrm{K\Omega}$

Current in $R_1 = I = \frac{V_{th}}{R_t} = \frac{7.5}{5} = 1.5\ \mathrm{mA}$

# EXERCISE # 02
## PART-A

Give the correct answers.

1. An electric circuit, whose characteristics or properties changed with the direction of its operation is called:
   (a) Bilateral circuit (b) Unilateral circuit
   (c) Active circuit (d) Passive circuit
2. Single closed path of an electric network is called:
   (a) Mesh (b) Node
   (c) One-port network (d) Two-port network
3. The circuit which have permanent parameters and not changed w.r.t current and voltage:
   (a) Active (b) Passive (c) Linear (d) Non-linear
4. The circuit which have not permanent parameters and changed w.r.t current and voltage:
   (a) Active (b) Passive (c) Linear (d) Non-linear
5. Kirchhoff's first law is also called as:
   (a) Mesh law (b) Point law
   (c) Voltage law (d) Thevenin Theorem
6. According to KCL:
   (a) Algebraic sum of all currents meeting at a point is zero
   (b) Algebraic sum of all currents meeting at a point is not zero
   (c) Algebraic sum of all currents meeting at a point is one
   (d) All of these
7. It is a circuit which contains one or more EMF sources is called:
   (a) Active (b) Passive (c) Linear (d) Non-linear
8. Such junction or point in a circuit where two or more circuit elements are connected:
   (a) Node (b) Branch (c) Mesh (d) Loop
9. That part of a network, which lies between two junctions, is called:
   (a) Node (b) Branch (c) Mesh (d) Loop
10. It is a closed path, where there is only one path for passing the current:
    (a) Mesh (b) Loop (c) Branch (d) Both a & b
11. In any electric network, the current coming towards a point is equal to outgoing current from that point is called:
    (a) Point law (b) Mesh law
    (c) Super position theorem (d) Both a & b
12. According to Kirchhoff's law in any closed circuit the sum of IR drops and EMF is:
    (a) Zero (b) More than zero (c) Positive (d) Negative

13. The resistive drop in a mesh due to current flowing in clockwise direction must be taken as:
    (a) Negative drop (b) Positive drop
    (c) Both a & b (d) None of these
14. The part of a network between two nodes is called:
    (a) Branch (b) Node (c) Mesh (d) Circuit
15. The circuit having energy source is called:
    (a) Passive circuit (b) Linear circuit
    (c) Active circuit (d) Non-linear circuit
16. Ideal voltage source should have;
    (a) Zero Internal Resistance
    (b) Infinite Internal Resistance
    (c) Large value of e.m.f. (d) Low value of current
17. Kirchhoff's laws are valid for:
    (a) Linear circuits only (b) Passive time invariant circuits
    (c) Nonlinear circuits only circuits (d) Both linear and non linear
18. Super position theorem is not applicable for:
    (a) Voltage Calculation (b) Bilateral elements
    (c) Power calculations (d) Passive elements

**ANSWER KEY**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | b | 2. | a | 3. | c | 4. | d | 5. | b |
| 6. | a | 7. | a | 8. | a | 9. | b | 10. | a |
| 11. | a | 12. | a | 13. | b | 14. | a | 15. | c |
| 16. | a | 17. | a | 18. | c | | | | |

# PART-B

Give the short answers of the following questions.

1. State Kirchhoff's current law.
2. State Kirchhoff's voltage law.
3. What is meant by node?
4. Define the passive circuit?
5. Define the active circuit?
6. What do you mean by branch?
7. What is loop?
8. State in your own words the maximum power transfer theorem for a D.C circuit.
9. State in your own words, the super position theorem?
10. State in your own words, the Thevenin's theorem?
11. What is Mesh analysis?
12. What is difference between loop current and branch current?
13. How does a mesh differ from a loop?
14. What do you mean by the term Open circuit and short circuit?

15. What is the difference between an electric network and an electric circuit?

## PART-C

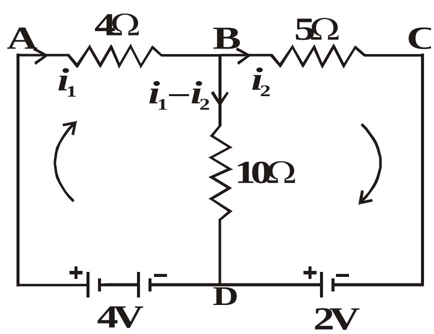
Give the detailed answers of the following questions.

1. State Kirchhoff's Laws. Elaborate with examples.
2. Write the steps used to solve a circuit by Kirchhoff's voltage law.
3. Explain super position theorem
4. Explain Thevenin Theorem.
5. What is meant by maximum power transfer theorem?

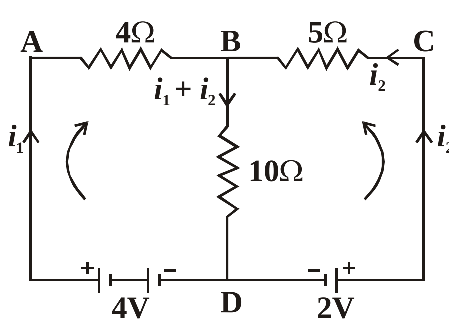
## PART-D

**Solve the following Problems**

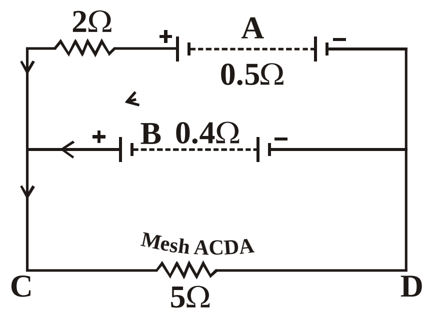
**Q.1 In the circuit shown, find the magnitude and direction of the current in each resistor. Internal resistance of cells negligible.**




**Q.2 In the circuit shown find the magnitude and direction of the current in each resistor. Internal resistance of cells negligible.**



**Q.3** **Two batteries, A and B, are connected to the circuit shown.**
**Battery A, e.m.f 100 volts.**
**Internal resistance 0.5 ohm.**
**Battery B, e.m.f. 80 volts**
**Internal resistance 0.4 ohm.**
**Find the current flowing in each battery and in 5-ohm resistor.**



# Chapter# 3

# WORK, POWER & ENERGY

## 3.1.1 WORK:

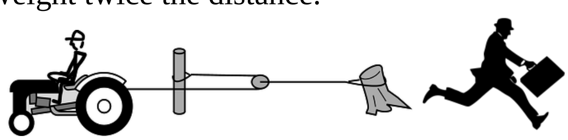
A force is said to do work when it acts on a body, and there is a displacement of the point of application in the direction of the force. For example, when you lift a suitcase from the floor, the work done on the suitcase is the force it takes to lift it (its weight) times the height that it is lifted.

The SI unit of work is the newton-meter or joule (J).

The work done by a constant force of magnitude F on a point that moves a displacement (not distance) s in the direction of the force is the product,

$$W = F.S$$

For example, if a force of 10 newton's (F = 10 N) acts along a point that travels 2 meters (s = 2 m), then it does the work W = (10 N) (2 m) = 20 N m = 20 J. This is approximately the work done lifting a 1 kg weight from ground to over a person's head against the force of gravity. Notice that the work is doubled either by lifting twice the weight the same distance or by lifting the same weight twice the distance.



## 3.1.2 ELECTRICAL POWER:

Electric power is usually produced by electric generators, but can also be supplied by sources such as electric batteries. Electric power is generally supplied to businesses and homes by the electric power industry.

Electric power is the rate at which electric energy is transferred by an electric circuit. The SI unit of power is the watt, one joule per second. In other words it is the rate of doing work, measured in watts, and represented by the letter P. The term wattage is used colloquially to mean "electric power in watts." The electric power in watts produced by an electric current consisting of a charge of Q coulombs every t seconds passing through an electric potential (voltage) difference of V is;

$$P = \text{work done per unit time} = \frac{QV}{t} = IV$$

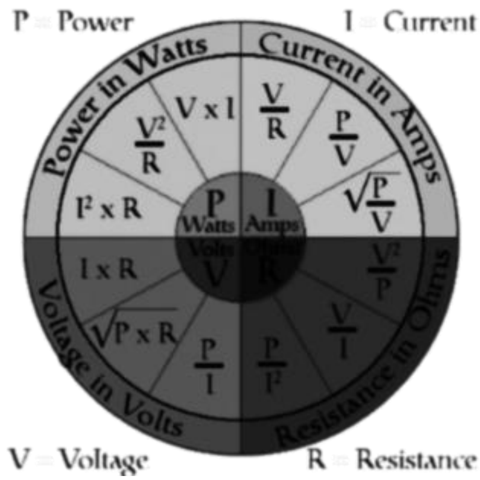
Where;

Q is electric charge in coulombs, t is time in seconds, I is electric current in amperes and V is electric potential or voltage in volts

In the case of resistive (Ohmic, or linear) loads, Joule's law can be combined with Ohm's law (V = I·R) to produce alternative expressions for the dissipated power:

$$P = I^2R = \frac{V^2}{R}$$

Where; R is the electrical resistance.




## 3.1.3 MECHANICAL POWER:

Power in mechanical systems is the combination of forces and movement. In particular, power is the product of a force on an object and the object's velocity, or the product of a torque on a shaft and the shaft's angular velocity. It is usually measured in horse power. (One horse power = 746 watt)

## 3.1.4 ENERGY:

Energy is a property of objects, transferable among them via fundamental interactions, which can be converted in form but not created or destroyed. The capacity of a body to do a work is called energy. The joule is the SI unit of energy, based on the amount transferred to an object by the mechanical work of moving it 1metre against a force of 1 Newton.

The joule is the International System of Units (SI) unit of measurement for energy. It is a derived unit of energy, work, or amount of heat. It is equal to the energy expended (or work done) in applying a force of one newton through a distance of one meter.

However energy is also expressed in many other units such as ergs, calories, British Thermal Units, kilowatt-hours and kilocalories for instance. There is always a conversion factor for these to the SI unit; for instance; one kWh is equivalent to 3.6 million joules. The CGS energy unit is the erg, and the imperial and US customary unit is the foot pound.

## 3.2 CONVERSION OF ELECTRICAL ENERGY TO MECHANICAL ENERGY:

In electrical motor electrical energy is converted into mechanical energy. Electrical input is given to motor, and mechanical energy is available at shaft of the motor. The theoretical principle behind production of mechanical force by the interactions of an electric current and a magnetic field, Ampère's force law, was discovered later by André-Marie Ampère in 1820. The conversion of electrical energy into mechanical energy by electromagnetic means was demonstrated by the British scientist Michael Faraday in 1821. A free-hanging wire was dipped into a pool of mercury, on which a permanent magnet (PM) was placed. When a current was passed through the wire, the wire rotated around the magnet, showing that the current gave rise to a close circular magnetic field around the wire.

This motor is often demonstrated in physics experiments, brine substituting for toxic mercury. Though Barlow's wheel was an early refinement to this Faraday demonstration, these and similar homopolar motors were to remain unsuited to practical application until late in the century.

One Joule of energy is expended electrically when one coulomb is moved through a potential difference of one volt, and 4187 joules of mechanical or electrical energy, if wholly converted into heat, will raise the temperature of one kilogram of water by 1ºC, called one Kilo Calorie.

The work done in moving a quantity of electricity Q coulombs through a potential difference of V volts is VQ joules.

$$W=VQ$$

If current I amperes flows through a circuit of resistance R ohms for time t seconds, then work done or electrical energy expanded = VQ joules, where V=IR is the volt drop across the circuit.

But Q $= It$

Energy expanded $= VIt$ Joules

$= I^2 Rt$ Joules

$= \frac{V^2 t}{R}$ Joules

In foot pound second unit of power is horse power. It is the rate of doing work of 33000 ft. lb per minute, i.e. 550 Ft. lb per second.

1 HP = 550 Ft. lb per second

= 550 x 1.356 Joules per second

= 746 Joules per second

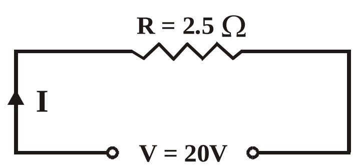
= 746 Watt

1 KW = 1.34 HP

## EXAMPLE–3.3.1:

**A potential difference of 20 volts is applied across a resistor of 2.5 Ohm. Calculate the current and the power dissipated.**

## Solution:




$R = 2.5\ \Omega \quad V = 20\ V$

$I = ? \quad$ Power (P) = ?

$$I = \frac{V}{R} = \frac{20}{2.5} = 8A$$

$$P = VI = 20 \times 8 = 160 \text{ watt}$$

## EXAMPLE – 3.3.2:

A consumer consumes 1000 watts load per hour daily for one month. Calculate the total energy bill of the consumer if per unit rate is Rs.9. Take 1 month = 30 days.

## Solution:

P = 1000 Watt Time = 1 hour

Rate = Rs.9.00 Energy bill = ?

$$\text{Energy consumed} = \frac{\text{Watt} \times \text{Time in hour}}{1000} \times \text{days} = \frac{1000 \times 1}{1000} \times 30$$

$$= 30 \text{ kwh or } 30 \text{ units}$$

Energy bill or cost = Total units × Rate

$$= 30 \times 9 = \text{Rs. } 270$$

## EXAMPLE – 3.3.3:

An electric iron is marked 200-volt, 350 watts. What current does it take and its hot resistance? What is weekly cost of using it for 30 minutes daily at Rs. 2.00 per unit?

## Solution:

V = 200 volt    I = ?

P = 350 watt    R = ?

Time = 30 min. $= \frac{30}{60} = 0.5\text{hr.}$

Rate = Rs. 2 per unit

Cost = ?

$$I = \frac{P}{V} = \frac{350}{200} = 1.75\text{A}$$

$$R = \frac{V}{I} = \frac{200}{1.75} = 114.3\ \Omega$$

$$\text{Energy consumed per week} = \frac{\text{Watt} \times \text{Hours}}{1000} \times \text{days} = \frac{350 \times 0.5}{1000} \times 7$$

$$= 1.225\ \text{KWh or Units}$$

$$\text{Cost} = \text{Units} \times \text{Rate (in rupees)}$$
$$= 1.225 \times 2 = \text{Rs. } 2.45\ \text{Ans.}$$

## EXAMPLE – 3.3.4:

Calculate the energy bill of the month of May of a house. The load connected and daily working hours of each load is as under. The rate is Rs. 2.00 per unit.

| LOAD | WORKING HOURS |
|---|---|
| 6 Lamps of 100 watts | 6 |
| 2 Tube Lights of 40 watt | 8 |
| 4 Fans of 150 watt | 5 |
| 1 Refrigerator of 100 watt | 18 |
| 1 Electric Iron of 1000 watt | 2 |

## Solution:

Energy consumed by 6 lamps = Number $\times$ Wattage $\times$ time

$= 6 \times 100 \times 6 = 3600$ Wh

Energy consumed by 2 tube lights $= 2\times40\times8 = 640$ Wh

Energy consumed by 4 fans $= 4 \times 150 \times 5 = 3000$ Wh

Energy consumed by 1 refrigerator $= 1\times100\times18 = 1800$ Wh

Energy consumed by 1 electric iron $= 1\times1000\times2 = 2000$Wh

Total energy consumed = 11040 Wh

Total energy consumed in one month:

$$= \frac{\text{Watt hours} \times \text{days}}{1000} \quad (\text{as the month of May} = 31 \text{ days})$$

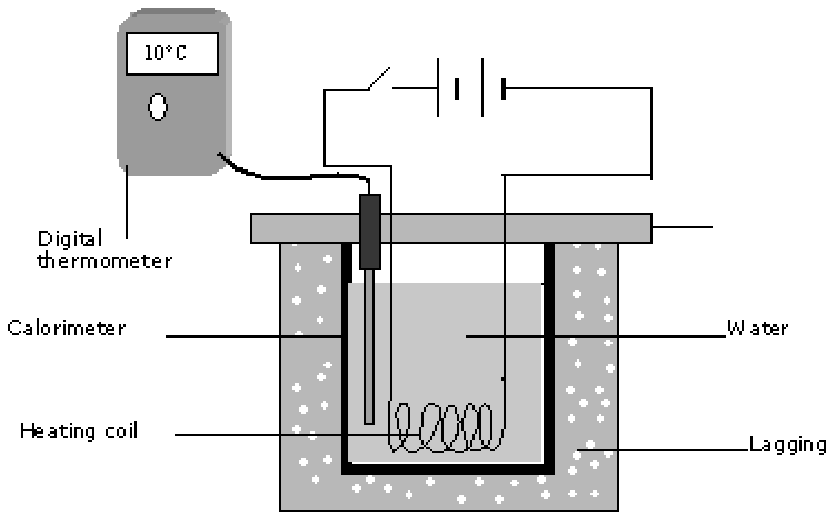
$$= \frac{11040 \times 31}{1000} = 342.24 \text{ KWh or units}$$

Total energy bill = Units $\times$ Rate (in Rupees)

$= 342.24 \times 2 =$ Rs. 684.48

## 3.4 HEATING EFFECT OF CURRENT:

The heating effect of electric current is used in many everyday devices. Electric cookers, kettles and toasters are among the household appliances that rely on it. Joule’s Law states that the rate at which heat is produced in a resistor is proportional to the square of the current flowing through it, if the resistance is constant. Heat is produced in current-carrying conductors, resulting in an increase in temperature of the conducting material. The heating is a result of the collisions between the moving free electrons and the relatively stationary atoms of the conductor material. As a result, heating increases rapidly with increase of current flow, since a greater rate of flow results in more collisions. Everyday examples of heating effect of electricity are the heating effect of an electric current has many practical applications, e.g. in radiant electric fires, cookers, hairdryers, kettles, toasters, domestic irons, immersion heaters, etc.




## 3.5 JOULE'S LAW:

Joule's law shows the relation between heats generated by an electric current flowing through a conductor. It is named after James Prescott Joule and shown as:

The rate at which heat is produced in a resistor is proportional to the square of the current flowing through it, if the resistance is constant.

$$Q = I^2 R t \text{ Joules}$$

Where Q is the amount of heat, I is the electric current flowing through a conductor, R is the amount of electric resistance present in the conductor, and t is the amount of time that this happens for.

Heat produced by heater = Q = Power x time (Seconds)

4187 Joules = 1 Kilo Calorie heat

$$\text{Heat produced by heater} = \frac{\text{Power(W)} \times \text{time (seconds)}}{4187} \text{ k Calorie}$$

Useful Heat = Heat produced – Heat loss in radiations

Heat required to heat up water up to required level = Weight of water x Rise in temperature

Heat required = Wt (Kg) x ( $T_2 - T_1$ ) K Calorie

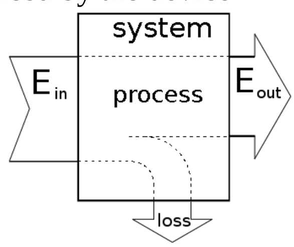
To heat water up to required level

Heat required = Useful heat

## 3.6 THERMAL EFFICIENCY:

In general, thermal efficiency is the ratio between the useful heat output of a device and the heat produced by the device.

$$\text{Efficiency} = \eta = \frac{\text{Useful heat output of device}}{\text{Heat produced by the device}}$$




## EXAMPLE – 3.7.1:

An electric kettle draws a current of 10 A when connected to the 230 V mains supply. Calculate (a) the power of the kettle (b) the final temperature of water, if 10 Kg of water having initial temperature of 15°C is to be heated up in 15 minutes. Take efficiency of kettle 75%.

**Solution;**

(a) P = IV = 10 × 230 = 2300 W = 2.3 kW.

Heat required=Weight of water×(Final temperature - Initial temperature)

= 10 × (T–15) K Calorie

$$\text{Heat produced} = \frac{\text{Power(W)} \times \text{time (seconds)}}{4187}$$

$$= \frac{2300 \times 15 \times 60}{4187} = 494.4 \text{ K Calorie}$$

$$\text{Useful heat} = \text{Heat produced} \times \frac{\eta}{100}$$

$$= 494.4 \times \frac{75}{100} = 370.8 \text{ K Calorie}$$

Heat required = Useful heat

10 X (T-15) = 370.8

$$T - 15 = \frac{370.8}{10} = 37.08$$

T = 37.08 + 15 = 52.08°C

## EXAMPLE – 3.7.2:

How long will it take to raise the temperature of 880 grams of water from16°C to boiling point? The heater takes 2 A at 220 V. Take efficiency of kettle 90%.

## Solution:

(a) $P = IV = 2\times 220 = 440$ W

Heat required=Weight of water × (Final temperature - Initial temperature)

$= 0.88 \times (100-16)$ K Cal $= 73.92$ K Cal

Heat produced $= \dfrac{\text{Power(W)} \times \text{time (seconds)}}{4187}$

$= \dfrac{440 \times t \times 60}{4187} = 6.305t$ K Cal

Useful heat $=$ Heat produced $\times \dfrac{\eta}{100}$

$= 6.305t \times \dfrac{90}{100} = 5.675t$ K Cal

Heat required = Useful heat

$5.675t = 73.92$

$t = \dfrac{73.92}{5.675} = 13.03$ minutes

## EXAMPLE – 3.7.3:

An electric kettle contains 1500 grams of water at 15°C. It takes 15 minutes to raise the temperature to 95°C. Assuming the heat losses due to radiation & heating the kettle to be 14kg calorie, find the current taken if the supply voltage is 100volts.

## Solution:

Weight of water = 1500 grams $= \dfrac{1500}{1000} = 1.5$ Kg

Time = 15min $= 15 \times 60 = 900$ sec

Heat losses = 14 K-calori    t1 = 15°C

Voltage = 100V    t2 = 95°C

Current = ?

Heat required = Weight of water (t2 – t1)

$= 1.5\ (95 - 15) = 120$ KCal.

$$\text{Heat produced} = \frac{V\ I\ t}{4187} = \frac{100 \times I \times 900}{4187} = 21.5 \times I \ \text{KCal.}$$

Heat produced = Heat required + Heat loss

$21.5\ I = 120 + 14 \Rightarrow 134$

$I = 134 / 21.5 = 6.2A$

## EXAMPLE – 3.7.4:

A heater contains 1600 grams of water at 20°C. it takes 12 minutes to raise the temperature to 100°C. assuming the losses due to radiation and heating the kettle to be 10Kg-calorie, find the power rating of the heater.

## Solution:

Weight of water = 1600grams = 1600/1000 = 1.6 Kg

Time = 12 min = 12 × 60 = 720 sec

$t_1 = 20°C$ heat losses = 10 KCal

$t_2 = 100°C$ power of heater , W = ?

Heat required = weight (t2 – t1)

$= 1.6\ (100 - 20) = 128$ KCal

$$\text{Heat produced} = \frac{W \times t}{4187} = \frac{Wx720}{4187} = 0.172 \times W \ \text{KCal}$$

Heat produced = Heat required + Heat loss

$0.172\ W = 128 + 10 \Rightarrow 138$

$$W = \frac{138}{0.172} = 802W$$

$W = 802 / 1000 = 0.8$ KW

# EXERCISE # 03

## PART-A

1. Product of force and distance is called:
   (a) Power (b) Work done (c) Speed (d) Velocity
2. The unit of work in MKS system is:
   (a) Watt (b) Ampere (c) Joule (d) Volt
3. Electrical unit of power is:
   (a) Watt (b) Ampere (c) Volt (d) Ohm
4. In D.C circuit V.I is used to find:
   (a) Current (b) Voltage (c) Flux (d) Power
5. Capacity to do the work is called:
   (a) Energy (b) Power
   (c) Resistance (d) Capacitance
6. Rate of doing work is called:
   (a) Energy (b) Power (c) Joule (d) Speed
7. 1 watt × 1 second =
   (a) 1 meter (b) 1 farad
   (c) 1 Joule (d) All of these
8. Trade unit of Electrical energy:
   (a) KWH (b) WH (c) H.P (d) B.H.P
9. 1 horse power is equal to:
   (a) 746W (b) 736W (c) 1000W (d) 646W
10. Formula of electrical energy is:
    (a) $W \times t$ (b) $I^2Rt$
    (c) $\frac{V^2}{R} \times R$ (d) All are correct
11. If lamp 100W is burn on 15 hours, then the energy consumes:
    (a) 0.5 unit (b) 1.5 unit
    (c) 6.67 unit (d) 15 unit
12. A potential difference of 20V is applied a resistor of 2.5 ohm. The power will be:
    (a) 170 watts (b) 180 watts
    (c) 190 watts (d) 160 watts
13. 746 watts are equal to:
    (a) 1 HP (b) 2 HP (c) 3 HP (d) 4 HP
14. In S.I units the unit of Heat is:
    (a) Ampere (b) Degree centigrade
    (c) Kelvin (d) Kilo-calorie
15. B.T.U stands for:
    (a) British Thermal Unit (b) Big Thermal Unit

(c) Both Thermal Unit (d) Bit Thermal Unit

16. The amount of heat energy to raise the temperature of 1 kg of water 1ºC is called:
    (a) Calorie (b) Kilo-calorie
    (c) Kelvin (d) B.T.U
17. 1 kilo calorie is equal to:
    (a) 3187 Joule (b) 4100 J
    (c) 4187 J (d) 1 B.T.U
18. Efficiency is ratio of:
    (a) $\frac{\text{Input}}{\text{Output}}$ (b) $\frac{\text{Output}}{\text{Input}}$
    (c) $\frac{\text{Output}}{\text{Losses}}$ (d) $\frac{\text{Input}}{\text{Losses}}$
19. Thermal efficiency is ratio of:
    (a) $\frac{\text{Heat actually utilized}}{\text{Total heat produced electrically}}$
    (b) $\frac{\text{Total heat produced electrically}}{\text{Heat actually utilized}}$
    (c) Heat actually utilizes × Total heat produced electrically:
    (d) All of them
20. Electrical energy bill is calculated with:
    (a) kWh × Rate/unit (b) Wh × Rate/unit
    (c) W sec. × Rate/unit (d) kW × Rate/unit

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | b | 2. | c | 3. | a | 4. | d | 5. | a |
| 6. | b | 7. | c | 8. | a | 9. | a | 10. | d |
| 11. | b | 12. | d | 13. | a | 14. | d | 15. | a |
| 16. | b | 17. | c | 18. | b | 19. | a | 20. | a |

## PART-B

Give the short answers of the following.

1. Give the SI Units of force, work, power of energy.
2. Give the relation between kWh, MJ and kilo-calorie.
3. Define work.
4. Define foot-pound
5. Define Watt.
6. Define Energy.
7. Define horse power.
8. What is kilo-calorie?

9. Define thermal efficiency.
10. Express Joule's law of electric heating.
11. Enumerate few applications of heating effect of electric current.
12. A potential difference of 20 volts is applied across a resistor of 2.5 ohm. Calculate the current of power dissipated.
13. A D.C generator supplies 250 amperes at 220 Volts. What is its power output in watts and in H.P?
14. What is trade unit of electrical energy?
15. Write the formula L for finding power in DC circuit.

## PART-C

Give the detailed answers of the following questions.

1) Define Work, Power and Energy and write down Unit of each one.
2) What is meant by energy, Explain? Moreover, how an electricity bill for a house is calculated?
3) Explain heating effects of current, moreover state the Joule's law.
4) Explain thermal efficiency with the help of an example.
5) Explain Joule's law of electric heating. Give practical applications of heating effect of electric current.
6) How an electricity bill of a building calculated? Explain it.

## PART-D

## SOLVE THE FOLLOWING NUMERICAL PROBLEMS:

1- Find the hot resistance of a 220volt, 100-watt lamp and the cost of running it for 20 hours at Rs.2.00 per unit.

2- A motor takes an average current of 32 A at 440 V. Find the power absorbed in KW. How many B.O.T. units are used in 5 hours?

3- Find the hot resistance of 200 volt, 100watt lamp and the cost of running it for 20 hours at Rs.2.00 per unit. Find the hot resistance of 200 volt, 100watt lamp and the cost of running it for 20 hours at Rs.2.00 per unit.

4- A house load consists of 4 lamps of 60 W each, 4 tube lights of 40 W each, 4 fans of 150 W each, all load works 4 hours daily. Find the total energy bill of the house for the month of June. The rate per unit is Rs. 1.50.

Ans: (Rs. 180.)

5- Calculate the total number of units consumed in operating a100-watt lamp 5 hours, 500 watts electric iron for 3 hours and 1 kW heater for 4 hours. Find also the cost of energy consumed if energy is supplied at the rate of Rs. 4.50 per unit.

Ans: (6units, Rs.27)

6- The details of electric load in a house are as follow:

(a) -3 lamps of 60 watt each for 5 hours per day.

(b) -2 fluorescent tubes of 40 W each for 4 hours per day.

(d) - 1 electric iron of 1 kW for 2 hours per day.

Determine the energy consumed and cost of energy at the rate of Rs. 4.25 per kWh for a month of 30 days.

Ans: (186.6 kWh, Rs.793.07)

7- An electric kettle is required to raise the temperature of ½ gallon of water from $20^0$c to $100^0$c in 15 minutes. Calculate the resistance of the heating element if the kettle is to be used on a 240 volts supply. Assume an efficiency of 80 percent.

Ans. (54.66Ω)

8- Find the time taken to boil 576 grams of water by means of electric heater whose efficiency is 80% . The heater is rated 0.5 kW and the initial temperature of water is $11^0$ c.

Ans. (8.9 min.)

9- A heater element of an electric kettle has a constant resistance of 100 Ω. The applied voltage is 240 volts and it is found that it takes 27.3 minutes to raise the temperature of ½ gallon of water from $20^0$c to boiling point. What is the thermal efficiency of the kettle?

Ans. (80.5% )

# Chapter # 4

## MAGNETIC EFFECTS

## OF ELECTRIC CURRENT

## 4.1 LAWS OF MAGNETIC FORCE:

Coulombs was the first to determine the laws of magnetic force. The laws states that the force acting between two magnetic poles is;

1- directly related to the pole strengths
2- inversely related to the square of the distance between them
3- Inversely proportional to the absolute permeability of the surrounding medium.

If $m_1$ and $m_2$ represent the magnetic strengths of the two poles, r is the distance between them and $\mu_0$ is the absolute permeability of the surrounding medium (vacuum), then the force F is given by

$$F \quad \alpha \quad \frac{m_1\, m_2}{\mu_0 r^2}$$

$$F = k\,\frac{m_1\, m_2}{\mu_0 r^2}$$

In SI units, the value of the constant k is $\frac{1}{4\pi}$

$$\therefore F = k\,\frac{m_1\, m_2}{\mu_0 r^2}$$

$$= \frac{m_1\, m_2}{4\pi\mu_0 r^2}\ \text{N} \qquad \text{in air}$$

If $\mu_r$ is relative permeability of another medium

$$= \frac{m_1\, m_2}{4\pi\mu_0\mu_r r^2}\ \text{N} \qquad \text{in other medium}$$

### 4.2.1 ABSOLUTE PERMEABILITY:

Many materials can encourage the development of a magnetic field within it. This term permeability is used in electromagnetism field. If any magnetic or non-magnetic material is subjected to a magnetic field then it will get some amount of magnetization automatically. The amount of magnetization of a material when it is subjected to a magnetic field is called as the permeability of that particular material.

The term absolute permeability refers to the permeability of a material when the surrounding medium is free space (vacuum). The standard value of the absolute permeability is equal to $4\pi \times 10^{-7}$ Henry per meter. So, from the definition absolute permeability of a material = B / H. Where, B is the magnetic flux density & H is the magnetic field. Being a standard value, absolute permeability is also known as the magnetic constant of free space and denoted by $\mu_0$.

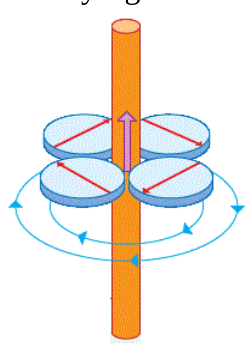
### 4.2.2 RELATIVE PERMEABILITY:

The term relative permeability refers to the permeability of a material when the surrounding medium is other than free space (vacuum), and denoted by $\mu_r$. It is equal to the ratio of the flux density produced in that material to the flux density produced in vacuum by the same magnetizing force.

$$\mu_r = \frac{B\ (material)}{B_0\ (Vacuum)} \text{ for same H}$$

## 4.3 MAGNETIC FIELD DUE TO STRAIGHT CURRENT CARRYING CONDUCTOR:

In the figure it is shown that an infinitely long straight conductor is carrying a current I in the upward direction. The magnetic field of such a conductor consists of circular lines of force having their plane perpendicular to the conductor and their centers at the center of the conductor. The direction of line of force can be determined by right hand rule.



Let the field at a point where a N - pole is placed as shown in the figure, is H and the distance of that point from the center of the conductor is r. Field strength at that point is H - means the N-pole wile experience a force of H Newton at that point. The direction of this force will be tangential to the circular force line. Now if the N pole is moved one round the conductor against this force, then work done is = Force × distance.

W = H × 2πr Joules ........................(i)

Again, it can be proved that when a unit N-pole takes a round of a current carrying conductor, the work done is numerically equal to the current flowing through the conductor. That means

$W = i$ Joules ..........................(ii)

From, equation (i) and (ii) we get,

$H \times 2\pi r = I$

So, $H = i / 2\pi r$,

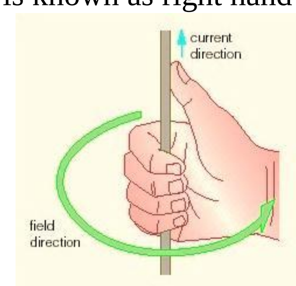
So, $B / \mu_0 = i / 2\pi r$

[Since, $B / \mu_0 = H$ in air]

=> $B = \mu_0 i / 2\pi r$ Weber per $m^2$ in air

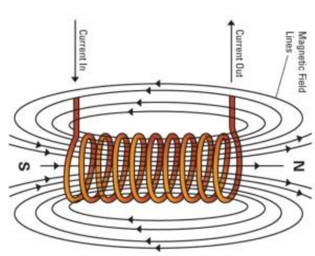
This is the expression of flux density at any point in the magnetic field due to straight current carrying conductor.

A convenient way of finding the direction of magnetic field associated Imagine that you are holding a current-carrying straight conductor in your right hand such that the thumb points towards the direction of current. Then your fingers will wrap around the conductor in the direction of the field lines of the magnetic field. This is known as right hand thumb rule.

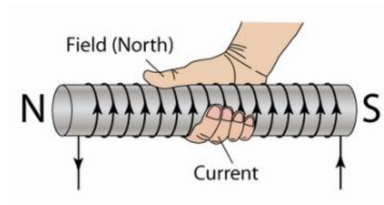



## 4.4 MAGNETIC FIELD OF A COIL:

An electric current flowing in a wire creates a magnetic field around the wire, due to Ampere's law (see figure below). To concentrate the magnetic field, in an electromagnet the wire is wound into a coil with many turns of wire lying side by side. The magnetic field of all the turns of wire passes through the center of the coil, creating a strong magnetic field there. A coil forming the shape of a straight tube (a helix) is called a solenoid.



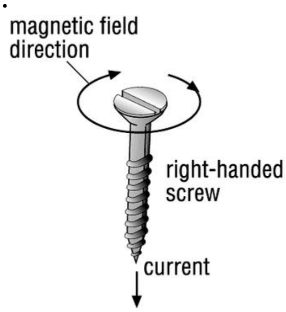
The direction of the magnetic field through a coil of wire can be found from a form of the right-hand rule. If the fingers of the right hand are curled around the coil in the direction of current flow (conventional current, flow of positive charge) through the windings, the thumb points in the direction of the field inside the coil. The side of the magnet that the field lines emerge from is defined to be the North Pole.




## 4.5 CORK SCREW RULE:

This rule is used for the determining the direction of magnetic field, If we rotate screw in the cork by a screwdriver. The current direction is same as the direction of screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field.

While electric current flows through a conductor, one magnetic field is induced around it. This can be imagined by considering numbers of closed magnetic lines of force around the conductor. The direction of magnetic lines of force can be determined by Maxwell's corkscrew rule or right-hand grip rule. As per these rules, the direction of the magnetic lines of force (or flux lines) is clockwise if the current is flowing away from the viewer, which is if the direction of current through the conductor is inward from the reference plane as shown in the figure.

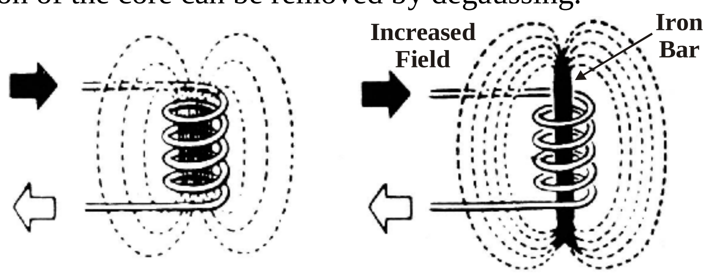



## 4.6 EFFECT OF IRON CORE IN A COIL:

Much stronger magnetic fields can be produced if a "core" of ferromagnetic material, such as soft iron, is placed inside the coil. The ferromagnetic core increases the magnetic field to thousands of times the strength of the field of the coil alone, due to the high magnetic permeability μ of the ferromagnetic material. This is called a ferromagnetic-core or iron-core electromagnet.

An electromagnet is a type of magnet in which the magnetic field is produced by electric current. The magnetic field disappears when the current is turned off. Electromagnets are widely used as components of other electrical devices, such as motors, generators, relays, loudspeakers, hard disks, MRI machines, scientific instruments, and magnetic separation equipment, as well as being employed as industrial lifting electromagnets for picking up and moving heavy iron objects like scrap iron.

The main advantage of an electromagnet over a permanent magnet is that the magnetic field can be rapidly manipulated over a wide range by controlling the amount of electric current. However, a continuous supply of electrical energy is required to maintain the field. When the current in the coil is turned off, most of the domains lose alignment and return to a random state and the field disappears. However, some of the alignment persists, because the domains have difficulty turning their direction of magnetization, leaving the core a weak permanent magnet. This phenomenon is called hysteresis and the remaining magnetic field is called permanent magnetism. The residual magnetization of the core can be removed by degaussing.




(a) Air Core Coil (b) Iron Core Coil

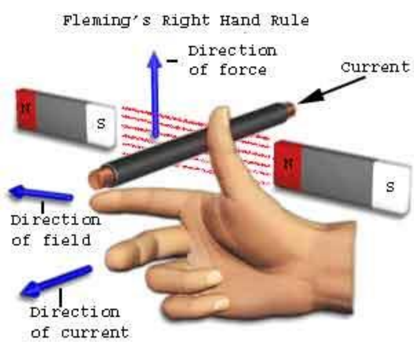
## HOW THE IRON CORE WORKS:

The material of the core of the magnet (usually iron) is composed of small regions called magnetic domains that act like tiny magnets. Before the current in the electromagnet is turned on, the domains in the iron core point in random directions, so their tiny magnetic fields cancel each other out, and the iron has no large-scale magnetic field. When a current is passed through the wire wrapped around the iron, its magnetic field penetrates the iron, and causes the domains to turn, aligning parallel to the magnetic field, so their tiny magnetic fields add to the wire's field, creating a large magnetic field that extends into the space around the magnet. The larger the current passed through the wire coil, the more the domains align, and the stronger the magnetic field is. Finally all the domains are lined up, and further increases in current only cause's slight increase in the magnetic field: this phenomenon is called saturation.

### 4.7.1 FLEMING'S RIGHT HAND RULE:

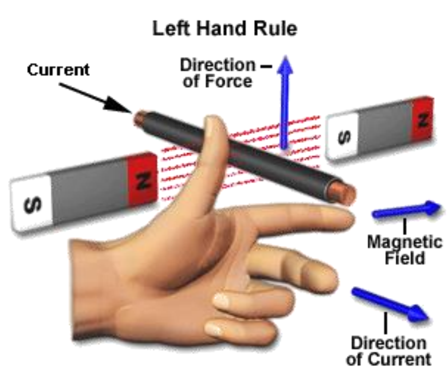
As per Faraday's law of electromagnetic induction, whenever a conductor moves inside a magnetic field, there will be an induced current in it. If this conductor gets forcefully moved inside the magnetic field, there will be a relation between the direction of applied force, magnetic field and the electric current. This relation among these three directions is determined by Fleming Right Hand rule. This rule is used to find direction of emf generated in generator.

This rule states "Hold out the right hand with the first finger, second finger and thumb at right angle to each other. If forefinger represents the direction of the line of force, the thumb points in the direction of motion or applied force, then second finger points in the direction of the induced current.




### 4.7.2 FLEMING LEFT HAND RULE:

Hold out your left hand with forefinger, second finger and thumb at right angle to one another. If the fore finger represents the direction of the field and the second finger that of the current, then thumb gives the direction of the force. This rule is used to find the direction of rotation of motor.



## 4.8 MECHANICAL FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD:

Whenever a current carrying conductor is placed in a magnetic field, it Experiences a mechanical force. This force is directly proportional to the current flowing, I in the conductor in ampere and it is also directly proportional to the length of the conductor, L in meter and magnetic flux density, B in Weber/m- of the magnetic field. Hence, we can write

$F \propto i$

$F \propto L$

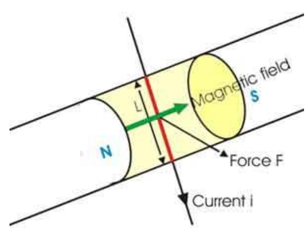
$F \propto B$

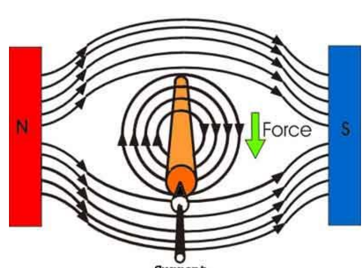
To compare the above factor we can write

$F \propto BIL$ Newton

It is found that whenever an electric current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the electric current and the magnetic field. In the figure it is shown that, a portion of a conductor of length L placed vertically in a uniform horizontal magnetic field strength H, produced by two magnetic poles N and S. If i is the electric current flowing through this conductor, the magnitude of the force acts on the conductor is,

$F = Bil$



# EXERCISE # 04

## PART-A

Give the correct answers.

1. Magnetic lines of force always enter from:
   (a) Horizontally (b) Vertically
   (c) N to S (d) S to N
2. Magnetic flux is indicated by:
   (a) q (b) F (c) $\phi$ (d) p
3. Total number of magnetic lines of force around a magnet is called:
   (a) Magnetic flux (b) Flux density (c) MMF (d) EMF
4. Unit of flux in M.K.S System:
   (a) Weber (b) Coulomb
   (c) Ampere turn (d) $Wb/m^2$
5. ................ is found between opposite poles of magnet:
   (a) Force of repulsion (b) Reluctance
   (c) Force of attraction (d) Flux density
6. ................ is found between same poles of magnet:
   (a) Force of repulsion (b) Reluctance
   (c) Force of attraction (d) Flux density
7. Flux per unit area in magnet is called:
   (a) Flux (b) Flux density
   (c) MMF (d) Ampere turns
8. The force which drives the flux in a magnetic circuit is called:
   (a) MMF (b) Flux density
   (c) EMF (d) Ampere turns
9. Formula of flux density is:
   (a) $\frac{A}{\phi}$ (b) $\frac{\phi}{A}$ (c) $\phi A$ (d) $\mu A$
10. SI unit of flux density is:
    (a) Weber (b) Weber/m
    (c) $Weber/m^2$ (d) Ampere turn
11. The property of a material which opposes the path of magnetic flux is:
    (a) Resistance (b) Reluctance
    (c) Impedance (d) Reactance
12. The force which drives the magnetic lines of force:
    (a) Flux density (b) MMF
    (c) EMF (d) Magnetizing force
13. Unit of Magneto motive force is:
    (a) $Wb/m^2$ (b) Ampere turn
    (c) Weber (d) Tesla
14. In a conductor or air core coil, force per unit length of a magnetic is:
    (a) Magnetizing force (b) Reluctance
    (c) Permiance (d) Flux density

15. Weber is the unit of:
(a) MMF (b) EMF (c) Flux (d) Flux density
16. Reluctance is indicated with:
(a) R (b) G (c) S (d) $\phi$
17. Reciprocal of reluctance is:
(a) Reluctivity (b) Permittivity
(c) Permiance (d) Permeability
18. The relation between flux density (B) and magnetizing force (H) is:
(a) $B = H^2$ (b) $H = B^2$ (c) $B = \mu H$ (d) $\mu = B/H$
19. The ratio between flux density (B) and magnetizing force (H) is:
(a) Permeability (b) Resistivity
(c) Reluctivity (d) Permittivity
20. Permeability is indicated with:
(a) P (b) $\mu$ (c) b (d) a
21. Formula of Permeability is:
(a) $\mu = B/H$ (b) $\mu = BH$
(c) $\mu = H/B$ (d) $\mu = B.A$
22. With cork screw rule, determine the direction of:
(a) Current (b) Power (c) Magnetic field (d) Both a & c
23. It is applied to determine the magnetic field in any coil:
(a) Right hand gripping rule (b) Faraday's law
(c) Coulomb's law (d) End rule
24. If a wire is wound in the shape of coil, and iron is inserted in it, it is called:
(a) Air core (b) Inductor
(c) Iron core (d) Both a & b
25. According to Fleming left hand rule, indicate:
(a) 1st finger, magnetic field (b) 2nd finger, current
(c) Thumb, direction of current (d) Both a & b

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | c | 3. | a | 4. | a | 5. | c |
| 6. | a | 7. | b | 8. | a | 9. | b | 10. | c |
| 11. | b | 12. | b | 13. | b | 14. | a | 15. | c |
| 16. | b | 17. | c | 18. | d | 19. | a | 20. | b |
| 21. | a | 22. | d | 23. | a | 24. | c | 25. | d |

# PART-B

Give the short answers of the following questions.
1. What is magnet?
2. Write the properties of magnet.
3. Define electro magnet.

4. What is magnetic field?
5. Define Weber.
6. Define flex density?
7. What do you mean by reluctance?
8. Define magneto-motive force.
9. What is ampere turns? Whose unit is this?
10. What is the power of magnetic field or magnetizing force? Write its formula.
11. State magnetic field around a straight current carrying conductor.
12. Define permeability.
13. What is permeability in free space.
14. Define the cork-screw rule.
15. State right hand gripping rule.
16. State Fleming's left-hand rule.
17. What is magnetic circuit?
18. Define the magnetic flux.

## PART-C

Give the detailed answers of following questions.

1) State laws of magnetic force.
2) Explain a force acting upon a current carrying conductor placed in a magnetic field.
3) Describe the effect of inserting of an iron core in a coil?
4) Differentiate the magnetic & electric circuit.
5) Explain magnetic field of a coil.

# Chapter # 5

## ELECTROMAGNETIC INDUCTION

## 5.1 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION:

Faraday was the first scientist who performed a number of experiments to discover the facts and figures of electromagnetic induction, he formulated the following law:

When magnetic flux changes through a circuit, an emf is induced in it which lasts only as long as the change in the magnetic flux through the circuit continues. The induced emf is directly proportional to the rate of change of magnetic flux through the coil. i.e.

emf induced / conductor = Flux cut per second

If B = Strength of the field in Wb/$m^2$

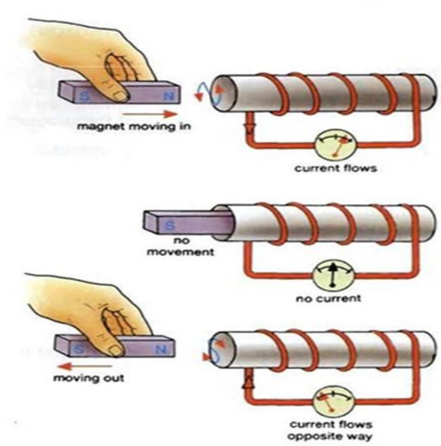
$\ell$ = Length of conductor (cutting the field) in m

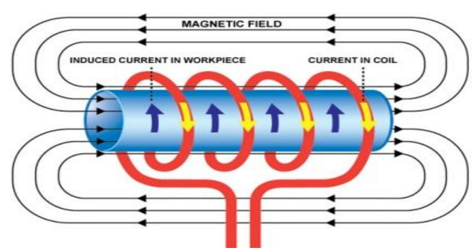
V = Velocity of movement in m/sec

Then emf induced / conductor = $B\ell v$ Volts

If N = number of turns in the coil.

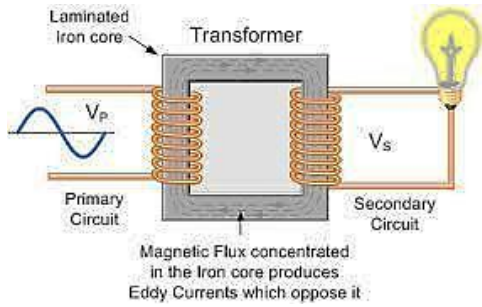
Then emf induced in coil = $B\ \ell vN$ Volts



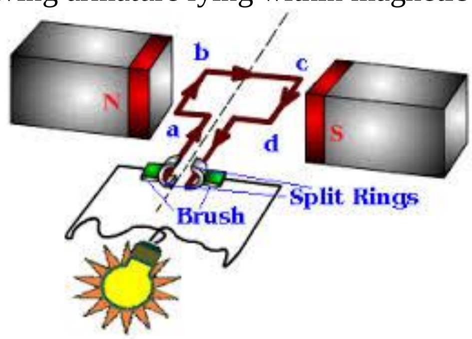

## 5.2.1 STATICALLYINDUCEDEMF:

The emf induced in a coil due to change of flux linked with it (change of flux is by the increase or decrease in current) is called statically induced emf. Transformer is an example of statically induced emf. Here the windings are stationary; magnetic field is moving around the conductor and produces the emf.




## 5.2.2 DYNAMICALLY INDUCED EMF:

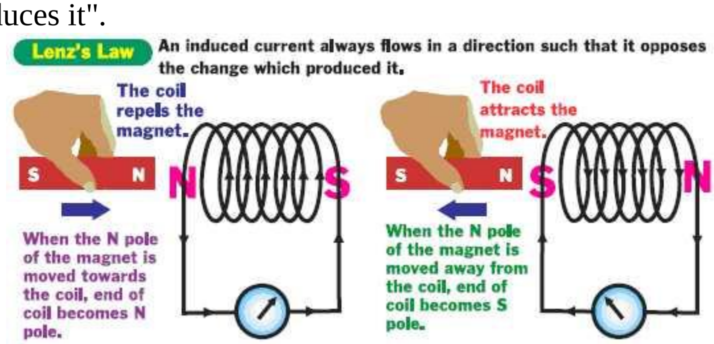
The emf induced in a coil due to relative motion of the conductor and the magnetic field is called dynamically induced emf, for example generator works on the principle of dynamically induced emf in the conductors which are housed in a revolving armature lying within magnetic field.




## 5.3 LENZ'S LAW:

Lenz's law describes that in order to produce an induced emf or induced current some external source of energy must be supplied otherwise no current will induce.Lenz's law states that"

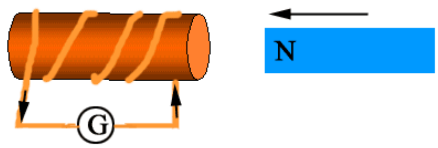
"The direction of induced current is always such as to oppose the cause which produces it".



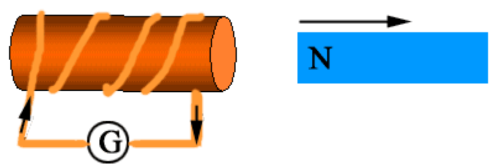
## EXPLANATION

Consider a bar magnet and a coil of wire.

When the N-pole of magnet is approaching the face of the coil, it becomes a north face by the induction of current in anticlockwise direction to oppose forward motion of the magnet.

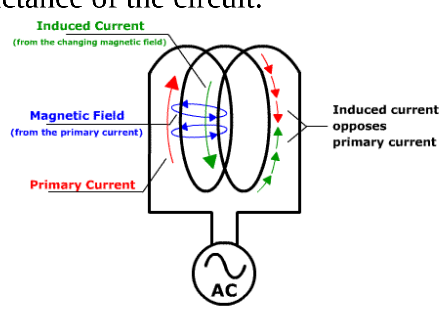



When the N-pole of the magnet is receding the face of the coil becomes a south pole due to a clockwise induced current to oppose the backward motion.




## 5.4 SELF-INDUCTANCE:

Inductance is a property of an electrical circuit whereby a change in the current flowing through the circuit induces an electromotive force (EMF) that opposes the change in current. In electrical circuits, any electric current ($i$) produces a magnetic field around the current-carrying conductor, generating total magnetic flux ($\Phi$) acting on the circuit. This magnetic flux tends to act to oppose changes in the flux by generating a voltage (a back EMF) that counters or tends to reduce the rate of change in the current. The ratio of the magnetic flux to the current is called the self-inductance, which is usually simply referred to as the inductance of the circuit.




The property of inductance is particularly noticeable and useful in the form of electrical devices known as inductors**.** An inductor is often a coil of wire wrapped around a material called the *core*. When an electric current is passed through the coil, a magnetic field is formed around it, and this field causes the inductor to resist changes in the amount of current passing through it. The quantitative definition of the self-inductance of a wire loop in SI units (Weber's per ampere) is

$$L = \frac{N\Phi}{i}$$

Where Φ denotes the magnetic flux through the area spanned by the loop, and N is the number of wire turns. The flux linkage $\lambda = N\Phi$ thus is

$$N\Phi = Li$$

If the current flowing around the circuit changes by an amount $dI$ in a time interval $dt$ then the magnetic flux linking the circuit changes by an amount $d\Phi = L\,dI$ in the same time interval. According to Faraday's law, an emf

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

is generated around the circuit. Since $d\Phi = L\,dI$, this emf can also be written

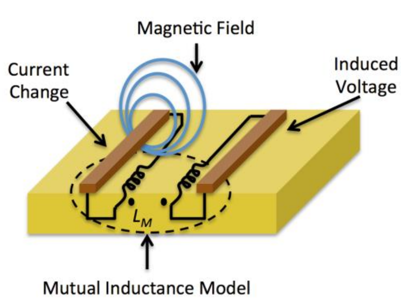
$$\mathcal{E} = -L\frac{dI}{dt}.$$

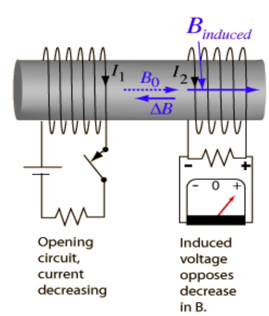
## 5.4.2 MUTUAL INDUCTANCE:

Mutual inductance occurs when the change in current in one inductor induces a voltage in another nearby inductor. It is important as the mechanism by which transformers work, but it can also cause unwanted coupling between conductors in a circuit. Mutual inductance is defined as the ratio of emf induced in the secondary coil to the rate of change of electric current in the primary coil.

$M = (emf)_s/\Delta I_p/\Delta t$

Value of 'M' depends upon the number of turns of secondary coil, their cross-sectional area and their closeness to each other. Unit of mutual inductance is Henry.




## 5.5 SYMBOLS AND UNITS:

The term 'inductance' was coined by Oliver Heaviside in February 1886. It is customary to use the symbol L for inductance, possibly in honor of the physicist Heinrich Lenz. In honor of Joseph Henry, the unit of inductance has been given the name **henry (H).**

1 H = 1 weber per ampere (Wb/A)

Inductance is a measure of the amount of EMF generated for a unit change in current. For example, an inductor with an inductance of 1 henry produces an EMF of 1 volt when the current through the inductor changes at the rate of 1 ampere per second.

# EXERCISE # 05

## PART-A

### Encircle the correct answer

1. Faraday states his law of electromagnetic induction in:
   (a) 1931 (b) 1934 (c) 1831 (d) 1840
2. When a conductor cuts the magnetic flux, an EMF is induced in that conductor:
   (a) Kirchhoff's law (b) Fleming left hand rule
   (c) Faraday's law (d) Coulomb law
3. The magnitude of the induced emf in a conductor is directly proportional to the rate of change of flux:
   (a) Directly (b) Indirectly
   (c) Directly square (d) Indirectly square
4. The magnitude of the induced emf in a conductor is directly proportional to the rate of change of flux, this is ......................
   (a) Coulomb's law (b) Fleming left hand rule
   (c) Faraday's law (d) Cork screw rule
5. When a conductor cut the magnetic flux then induce:
   (a) E.M.F (b) Current (c) Flux (d) Resistance
6. When magnetic field remain stationery and conductor cut the field then the emf induced:
   (a) Statically induced emf (b) Dynamically induced emf
   (c) Mutual induce emf (d) Self-induced emf
7. Inductance is denoted by:
   (a) M (b) W (c) Ohm (d) L
8. The unit of inductance is:
   (a) Henry (b) Ampere (c) Volt (d) Ohm
9. Induced emf is directly proportional to the rate of change of flux is called:
   (a) Kirchhoff Law (b) Laws of resistance
   (c) Faraday 2nd law of electromagnetic induction
   (d) Ohm's law
10. In Fleming right hand rule, the thumb shows
    (a) Direction of motion (b) Direction of flux
    (c) Electromagnetic induction (b) Direction of current
11. The working principle of all generators is:
    (a) Fleming left hand rule (b) Fleming right hand rule
    (c) Cork screw rule (d) Electromagnetic induction
12. In Fleming right hand rule, the first finger shows:
    (a) The direction of motion of conductor

(b) The direction of emf
(c) The direction of magnetic field
(d) The direction of current

13. To find the direction of motor we use.
(a) Ohm's law (b) Mutual induction law
(c) Faraday's law (d) Fleming left hand rule

14. Statically induced emf has types:
(a) 4 (b) 3 (c) 2 (d) 1

15. Transformer works on the principle of:
(a) Statically induced emf (b) Dynamically induced emf
(c) Electromagnetic induction (d) Mutual induction

16. The EMF generated by stationary the coil or conductor and vary the magnetic field is called:
(a) Statically (b) Dynamically
(c) Both (a) & (b) (d) All

17. Induced effect always opposes the cause that produced it:
(a) Faraday's law (b) Lenz's law
(c) Coulomb's law (d) Ohm's law

18. Stationary strong flux, ________ EMF in stationary conductors:
(a) No induced (b) Induced a little
(c) Induced high (d) Induced very high

19. EMF is induced only that time (moment), at which time flux ______:
(a) Linked with coil (b) Change
(c) Remains constant (d) Both b & c

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | c | 3. | a | 4. | c | 5. | a |
| 6. | b | 7. | d | 8. | a | 9. | c | 10. | a |
| 11. | b | 12. | c | 13. | d | 14. | c | 15. | a |
| 16. | a | 17. | b | 18. | a | 19. | b | | |

## PART-B

Give the short Answer.

1. State Faraday's first law of electromagnetic induction.
2. State Faraday's second law of electromagnetic induction.
3. State Fleming's right-hand rule.
4. Define dynamically induced e.m.f
5. Define statically induced e.m.f.
6. Write two types of statically induced e.m.f
7. Define mutual inductance.
8. Define self-inductance.
9. State Lenz's law.

10. If the same poles of 2 magnets brought to be near what will be happen?
11. Write differential form of formula for Faraday's second law of electromagnetic induction.
12. Draw a diagram of showing self-induced e.m.f.
13. Draw a simple circuit diagram of transformer
14. What is mutual induction.
15. Write the name of basic parts of transformer.
16. On which principle emf is induced in D.C generator?
17. Define mutually induced emf.

## PART-C

Give the answer in detail

1. State Faraday's Laws of Electromagnetic Induction.
2. Explain the Faraday's right-hand rule for electromagnetic induction.
3. Explain dynamically & statically induced emf.
4. Explain self and mutual induction.
5. Explain in detail Lenz's Law.

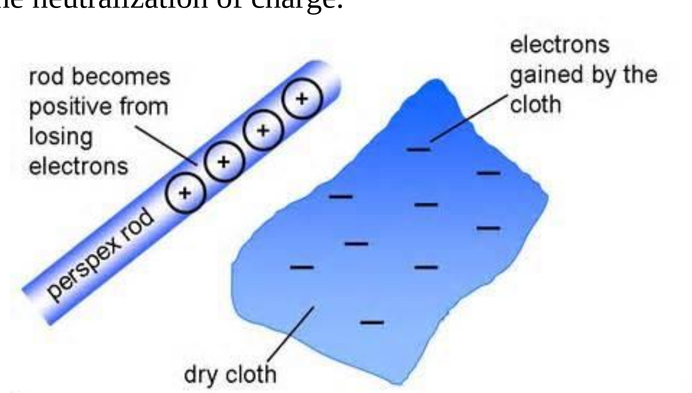
# Chapter # 6

## FUNDAMENTALS OF ELECTROSTATICS

## 6.1 STATIC ELECTRICITY:

Static electricity is an imbalance of electric charges within or on the surface of a material. The charge remains until it is able to move away by means of an electric current or electrical discharge. Static electricity is named in contrast with current electricity, which flows through wires or other conductors and transmits energy.

A static electric charge is created whenever two surfaces contact and separate, and at least one of the surfaces has a high resistance to electrical current (and is therefore an electrical insulator). The effects of static electricity are familiar to most people because people can feel, hear, and even see the spark as the excess charge is neutralized when brought close to a large electrical conductor (for example, a path to ground), or a region with an excess charge of the opposite polarity (positive or negative). The familiar phenomenon of a static shock–more specifically, an electrostatic discharge–is caused by the neutralization of charge.




## 6.2 ABOSOLUTE & RELATIVE PERMITTIVITY OF A MEDIUM:

While discussing electrostatic phenomenon, a certain property of the medium called its permittivity plays an important role. Every medium is supposed to possess two permittivity's.

1- Absolute permittivity ($\varepsilon_0$)

2- Relative permittivity ( $\varepsilon_r$ )

Absolute permittivity ( $\varepsilon_0$ )is the measure of the resistance that is encountered when forming an electric field in a vacuum or free space medium. In SI units, permittivity $\varepsilon_0$ is measured in farads per meter (F/m); and $\varepsilon_0 = 8.8541878176 \times 10^{-12}$F/m is the vacuum permittivity.

For measuring relative permittivity, vacuum or free space is choosen as a reference medium. Itis allotted an absolute permittivity of 8.8541878176 $\times 10^{-12}$F/m. Obviously, the relative permittivity of vacuum with reference to itself is unity, Hence for free space or vacuum;

Absolute permittivity = $\varepsilon_0$ = $8.8541878176 \times 10^{-12}$F/m

Relative permittivity = $\varepsilon_r$ = 1

Being a ratio of two similar quantities $\varepsilon_r$ has no units. Now take another medium. If its relative permittivity as compared to vacuum is $\varepsilon_r$, then its absolute permittivity is, $\varepsilon = \varepsilon_0\,\varepsilon_r$ F/m

If, for example, relative permittivity of mica is 5, then its absolute permittivity will be;

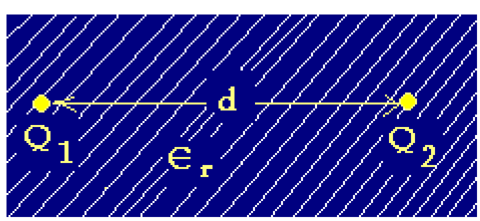
$\varepsilon = \varepsilon_0\,\varepsilon_r = 8.8541878176 \times 10^{-12} \text{ x } 5 = 44.27 \times 10^{-12}$ F/m

## 6.3 LAWS OF ELECTROSTATICS:

### 1ST LAW:

Like charges of electricity repel each other, whereas unlike charges attract each other.

### 2ND LAW:




According to this law, the force exerted between two point charges;

i) is directly proportional to the product of their strengths.

ii) is inversely proportional to the square of the distance between them.

iii) is inversely proportional to the absolute permittivity of the surrounding medium. This is known as Coulomb's Law of electrostatics

## 6.4 CAPACITANCE:

Capacitance is the ability of a body to store an electrical charge. Any object that can be electrically charged exhibits capacitance. A common form of energy storage device is a parallel-plate capacitor. In a parallel plate capacitor, capacitance is directly proportional to the surface area of the conductor plates and inversely proportional to the separation distance between the plates. If the charges on the plates are +q and −q respectively, and V give the voltage between the plates, then the capacitance C is given by

$$C = \frac{q}{V}.$$

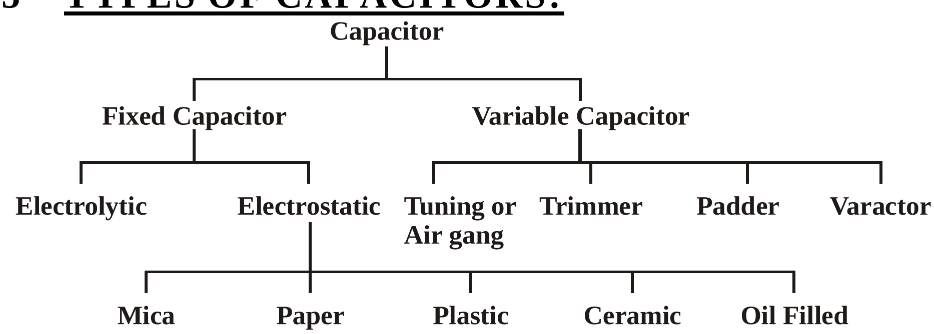
This gives the voltage/current relationship

$$I(t) = C\frac{\mathrm{d}V(t)}{\mathrm{d}t}.$$

The capacitance is a function only of the geometry (including their distance) of the conductors and the permittivity of the dielectric. It is independent of the potential difference between the conductors and the total charge on them.

The SI unit of capacitance is the farad (symbol: F), named after the English physicist Michael Faraday. A, 1 farad capacitor, when charged with 1 coulomb of electrical charge, has a potential difference of 1 volt between its plates. Historically, a farad was regarded as an inconveniently large unit, both electrically and physically. Its subdivisions were invariably used, namely the microfarad, Nano farad and Pico farad. More recently, technology has advanced such that capacitors of 1 farad and greater can be constructed in a structure little larger than a coin battery (so-called 'super capacitors'). Such capacitors are principally used for energy storage replacing more traditional batteries.

## 6.5 TYPES OF CAPACITORS:




**Note: According to course outline only the list of the types of capacitors is included and not detail is required.**

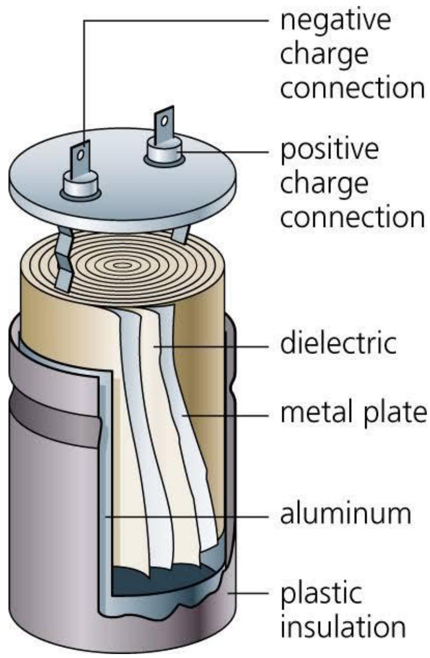
Capacitor has following two types according to working.

(1) Fixed or non-variable capacitor.

(2) Variable capacitor.

## (1) Fixed Capacitors

Such capacitor whose capacitance cannot be changed is called a fixed capacitor. Different components of these capacitors cannot move from their place due to which their capacitance remains fixed.




They have two types w.r.t dielectric and voltage.

(A) Electrostatic capacitors

(B) Electrolytic capacitors

**(A) Electrostatic Capacitors:**

Those capacitors which are made by using insulation e.g mica, paper and ceramic as a dielectric between two thin metallic plates are called electrostatic capacitors. Polarity is not considered while using these capacitors in circuits i.e. these can be used in any direction in the circuit.

They have following type w.r.t to dielectric materials.

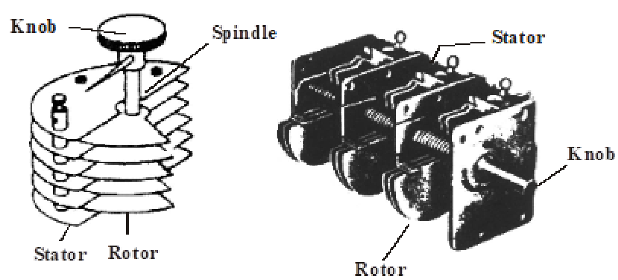
(i) Mica Capacitors
(ii) Paper Capacitor
(iii) Plastic Capacitors
(iv) Ceramic Capacitors
(v) Oil Filled Capacitors

### (B) Electrolytic Capacitors:

These capacitors have two types. In first type Aluminum oxide and in second type Tantalum oxide is used as dielectric. These capacitors are polarized. While using them much care is taken for polarity. Basically this capacitor consists of two foils or sheets of Aluminum or Tantalum. A gauze of insulating material is placed between them which is saturated with an electrolyte e.g. Aluminum Borate.

## (2) Variable Capacitors

Such capacitor whose capacitance or capacity can be changed is called a variable capacitor. Its capacitance varies by changing the distance between its plates or by changing the area of its plates. In these capacitors air, ceramic and mica is used as dielectric.




(i) Air Gange Capacitors

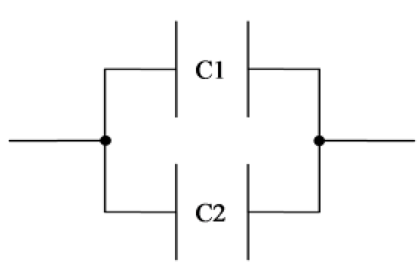
Their main types are as under.

(i) Tuning or Air gang Capacitors
(ii) Trimmers and Pedder's
(iii) Varactors

## 6.6 PROBLEM SOLVING OF CAPACITOR IN PARALLEL & SERIES CIRCUITS:

### A) CALCULATION OF CAPACITORS IN PARALLEL:

When capacitors are connected across each other (side by side) this is called a parallel connection. This is shown below.

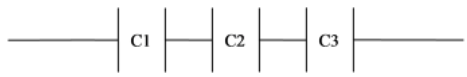


To calculate the total / overall capacitance of a number of capacitors connected in this way you add up the individual capacitances using the following formula:

$C_{Total} = C_1 + C_2 + C_3$ and so on

## 6.6.2 CAPACITORS IN SERIES:

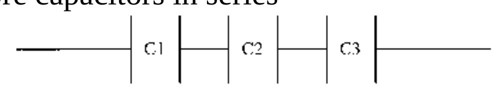
When capacitors are connected one after each other this is called connecting in series. This is shown below.




**a)** To calculate the total overall capacitance of two capacitors connected in this way you can use the following formula:

$$C\ Total = \frac{C1\ X\ C2}{C1 + C2}$$

**B)** Three or more capacitors in series




Calculate the total / overall capacitance of three or more capacitors connected in this way you can use the following formula:

$$\frac{1}{Ctotal} = \frac{1}{C1} + \frac{1}{C2} + \frac{1}{C3} +$$

## EXAMPLE - 6.6.2.1:

Calculate the total capacitance for three capacitors 10μF, 22μF, 4μF connected in parallel.

### Solution:

$C_1 = 10\mu F$ , $C_2 = 22\mu F$ , $C_3 = 4\mu F$

$C_T = C_1 + C_2 + C_3$

$C_T = 10 + 22 + 4 = 36\mu F$

## EXAMPLE - 6.6.2.2:

Calculate the total capacitance for three capacitors 0.1μF, 4.7μF, 2.2μF connected in parallel.

### Solution:

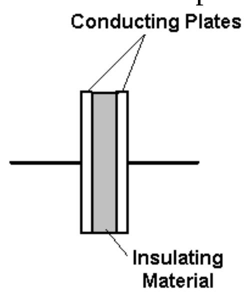
$C_1 = 0.1\mu F$ $C_2 = 4.7\mu F$ $C_3 = 4\mu F$

$C_T = C_1 + C_2 + C_3$

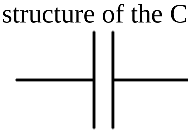
$C_T = 0.1 + 4.7 + 2.2 = 7\mu F$

## 6.7.1 <u>CHARGING A CAPACITOR</u>:

Capacitor is a passive device that stores energy in its Electric Field and returns energy to the circuit whenever required. A Capacitor consists of two Conducting Plates separated by an Insulating Material or Dielectric. The basic structure and the schematic symbol of the Capacitor are shown below.



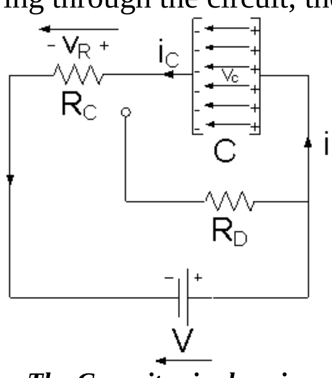

Basic structure of the Capacitor



Schematic symbol of the Capacitor

When a Capacitor is connected to a circuit with Direct Current (DC) source, two processes, which are called "charging" and "discharging" the Capacitor, will happen in specific conditions.

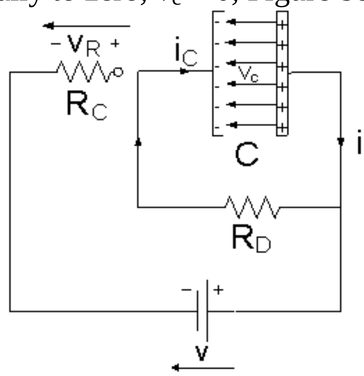
In Figure below, the Capacitor is connected to the DC Power Supply and Current flows through the circuit. Both Plates get the equal and opposite charges and an increasing Potential Difference, $v_c$, is created while the Capacitor is charging. Once the Voltage at the terminals of the Capacitor, $v_c$, is equal to the Power Supply Voltage, $v_c = V$, the Capacitor is fully charged and the Current stops flowing through the circuit, the Charging Phase is over.




***The Capacitor is charging***

A Capacitor is equivalent to an Open-Circuit to Direct Current, $R = \infty$, because once the Charging Phase has finished, no more Current flows through it. The Voltage $v_c$ on a Capacitor cannot change abruptly.

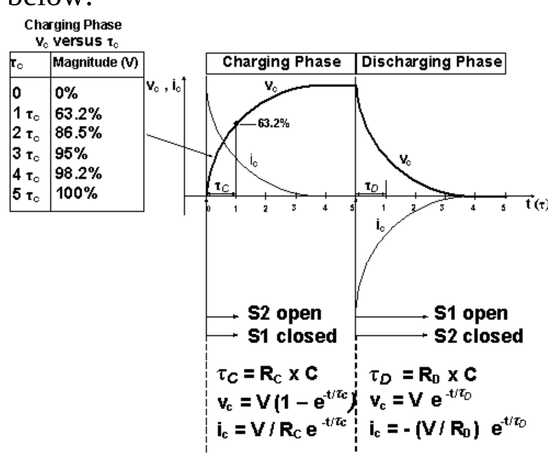
When the Capacitor disconnected from the Power Supply, the Capacitor is discharging through the Resistor $R_D$ and the Voltage between the Plates drops down gradually to zero, $v_c = 0$, Figure below.




***The Capacitor is discharging***

In Figures 3 and 4, the Resistances of $R_C$ and $R_D$ affect the charging rate and the discharging rate of the Capacitor respectively.

The product of Resistance R and Capacitance C is called the Time Constant $\tau$, which characterizes the rate of charging and discharging of a Capacitor, Figure below.




The Voltage $v_c$ and the Current $i_c$ during the Charging Phase and Discharging Phase

The smaller the Resistance or the Capacitance, the smaller the Time Constant, the faster the charging and the discharging rate of the Capacitor, and vice versa.

Capacitors are found in almost all electronic circuits. They can be used as a fast battery. For example, a Capacitor is a storehouse of energy in photoflash unit that releases the energy quickly during short period of the flash.

# EXERCISE # 06

## PART-A

**Chose the correct answer**

1. Electrostatic is branch of electricity concerned with,
   (a) Energy following across a gap between conductor
   (b) Charges at rest
   (c) Charges in motion (d) Energy in the form of charges
2. Same charges .............. each other:
   (a) Attract (b) Repel (c) Not effected (d) None
3. Opposite charges ................. each other:
   (a) Attract (b) Repel (c) Not effected (d) None
4. The ability of dielectric to concentrates its electric flux is called:
   (a) Permeability (b) Permittivity
   (c) Resistivity (d) Conductivity
5. Permittivity is indicated with:
   (a) $\varnothing$ (b) $\in$ (c) $\mu$ (d) $\Upsilon$
6. According to electrostatics law determine the formula of force between two charged bodies:
   (a) $K.\frac{Q_1Q_2}{d}$ (b) $K.\frac{Q_1Q_2}{d^2}$
   (c) $K.\frac{Q_1 + Q_2}{d}$ (d) $K.\frac{Q_1 + Q_2}{d^2}$
7. Capacitor, stores electric charge in:
   (a) Plates (b) Dielectric
   (c) Battery (d) Both a & b
8. Before charging, numbers of free electrons on both plates of the capacitor is:
   (a) Same (b) Different
   (c) Double (d) Any one
9. Capacitor charged with this source:
   (a) AC (b) DC
   (c) 1st AC, latter DC (d) Both a & b
10. Unit of capacitance is:
    (a) Farad (b) Micro Farad
    (c) Henry (d) Both a & b
11. The capacitance increased of capacitors connected in parallel plates:
    (a) With applied voltage
    (b) With decreasing area of plates

(c) With decreasing thickness of dielectric
(d) With increasing thickness of dielectric

12. Unit of energy saved in a capacitor is:
(a) Farad (b) Joule
(c) Coulomb (d) Henry

13. 1 Farad is equal to:
(a) $10^3\mu F$ (b) $10^6\mu F$ (c) $10^9\mu F$ (d) $10^{12}\mu F$

14. Energy store in capacitor is defined with this formula:
(a) $\frac{1}{2}QV$ (b) $\frac{1}{2}CV^2$ (c) $\frac{1}{2}.\frac{Q^2}{C}$ (d) All

15. The formula to determine capacitance:
(a) I/Q (b) Q/V (c) V/Q (d) QV

16. Total capacitance of different capacitors in parallel is equal to:
(a) $C_1 + C_2 + C_3$ (b) $\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
(c) $C_1 = C_2 = C_3$ (d) QV

17. With increasing capacitors in series, the total capacitance:
(a) Decreases (b) Increases
(c) Remains same (d) None

18. If different capacitors of same rating are connected in series, the total capacitance is equal to:
(a) $C_T = \frac{C}{n}$ (b) $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$
(c) $C_T = C_1 + C_2$ (d) Both a & b

19. Which answer is correct for the capacitors connected in series:
(a) $Q_T = Q_1 = Q_2$ (b) $V_T = V_1 = V_2$
(c) $C_T = C_1 + C_2$ (d) All

20. The potential difference across 10 micro farad capacitors charge it with 10mC is:
(a) 10V (b) 1kV (c) 1V (d) 100V

21. The charge on a 10 micro farad capacitor when the voltage applied to it is 10V.
(a) 100μC (b) 0.1μC (c) 0.01μC (d) 0.001μC

22. Four capacitors of 2mF are connected in parallel the equivalent capacitance is:
(a) 8μF (b) 0.5μF (c) 2μF (d) 6μF

23. Four capacitors 2mF are connected in series the equivalent capacitance is:
(a) 8μF (b) 0.5μF (c) 2μF (d) 6μF

24. Charge is denoted by:
(a) V (b) Q (c) C (d) T

25. The unit of capacitance is:
(a) Volt (b) Ampere (c) Henry (d) Farad
26. The material used in between two metallic plates of capacitor is called:
(a) Electric field (b) Electric flux
(c) Dielectric (d) All of these
27. If mica is used as dielectric then the capacitor will be:
(a) Mica capacitor (b) Paper capacitor
(c) Ceramic capacitor (d) Oil filled capacitor
28. The formula to find the capacitance is:
(a) C = QV (b) C = Q/V
(c) C = V/Q (d) All of these
29. State which of the following is true:
(a) c $\alpha$ a (b) c $\alpha$ d
(c) c $\alpha$ reflective permeability (d) c $\alpha$ 1/Q
30. The energy stored in a 10mF capacitor when charged to 500V is:
(a) 1.25μJ (b) 0.025μJ (c) 1.25J (d) 1.25C
31. When a voltage of 1kv is applied to a capacitor the charges onthe capacitor is 500mC. The capacitance of the capacitor is:
(a) 2pF (b) 0.5pF (c) 0.5μF (d) 0.5nF
32. A capacitor is charged 10,000mC. If the energy stored is 1J the voltage will be:
(a) 400v (b) 200v (c) 100v (d) 50v

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | b | 2. | b | 3. | a | 4. | b | 5. | b |
| 6. | b | 7. | a | 8. | a | 9. | b | 10. | d |
| 11. | c | 12. | b | 13. | b | 14. | d | 15. | b |
| 16. | a | 17. | a | 18. | a | 19. | a | 20. | c |
| 21. | a | 22. | a | 23. | b | 24. | b | 25. | d |
| 26. | c | 27. | a | 28. | b | 29. | a | 30. | c |
| 31. | c | 32. | b | | | | | | |

## PART-B

**Give the short answer of the following questions**

1. What static electricity?
2. Define permittivity.
3. State first law of electrostatic.
4. State second law of electro static.
5. What is electric field?
6. What is electric flux?
7. Define capacitance.
8. State the factors on which the capacitance depends upon.
9. Define Farad.

10. What is dielectric?
11. What is break down voltage.
12. What is the use of capacitor?
13. Write the types of fixed capacitor.
14. Write the types of variable capacitor.
15. If many capacitors are connected in series, what will be formula for total capacitance.
16. If three capacitors are connected in parallel, what will be formula for total capacitance.
17. The capacitor of 4, 6, & 5 μF are connected in parallel, find the total capacitance.
18. Differentiate b/w fixed & variable capacitor.

## PART-C

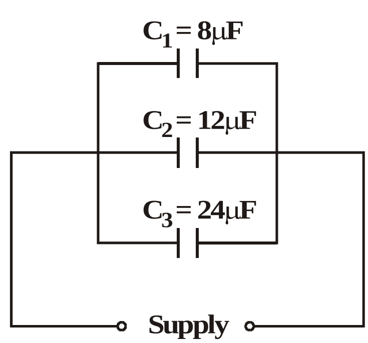
**Give the detailed answer of the following questions.**

1) What in meant by static Electricity? Explain in brief.
2) What are Laws of Electrostatics? Explain.
3) What is capacitor? Write down the names of the types of capacitors
4) Compare characteristics capacitors in series and parallel.
5) State the charging and discharging of a capacitor.

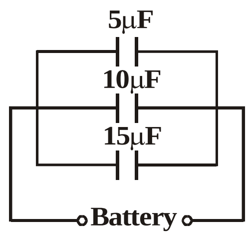
## PART-D

**SOLVE THE PROBLEMS**

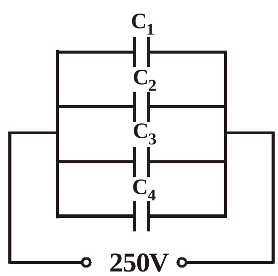
Q.1: Three capacitors of 8, 12 & 24 μF are connected in parallel. Find total capacitance of the circuit. (Ans. 44μF)



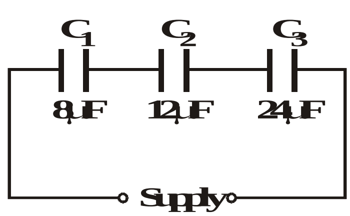
Q.2: Calculate the total capacitance of the circuit shown. (Ans. 30μF)



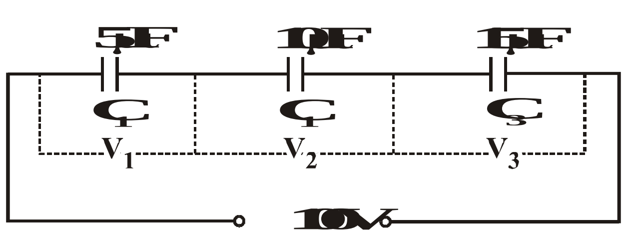

Q.3: Four capacitors of 750, 1000, 1500, 2000 μF are connected in parallel with supply of 250V. Find the total capacitance. (Ans. 5250μF)




Q.4: Three capacitors of 8, 12 & 24 μF are connected in series. Find the total capacitance of the circuit. (Ans. 4μF)

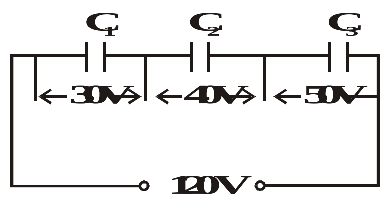



Q.5: Three capacitors of 5,10 and 15μF are connected in series across a 100-volt supply. Find the equivalent capacity. Find also the voltage across each.




Ans. $Q_T$ = 273μC , $V_1$ = 54.5 V , $V_2$ =27.3V , $V_3$ = 18.2V

Q.6: Three capacitors are connected in series across a 120-volt supply. The voltages across them are 30, 40 and 50 volts and the charge on each is 4500 micro coulombs. What is the value of each capacitor and of the series combination?




Ans. $C_T = 37.5\mu F$ , $C_1 = 150\mu F$ , $C_2 = 112.5\mu F$ , $C_3 = 90\mu F$

Q.7: Find the total capacitance of following capacitors connected in series.

(a) 10µF, 3.3µF (b) 47µF, 22µF (c) 33µF, 4.7µF.

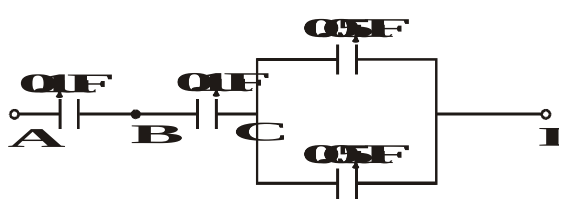
Ans. 2.48µF , 14.98µF , 4.11µF

Q.8: Calculate total capacitance of the following capacitors in series.

(a) 10µF, 10µF, 10µF (b) 10µF, 3.3µF, 2.2µF

Ans. 3.33µF , 1.17µF

Q.9: Determine the equivalent capacitance of series parallel combination of capacitors shown.

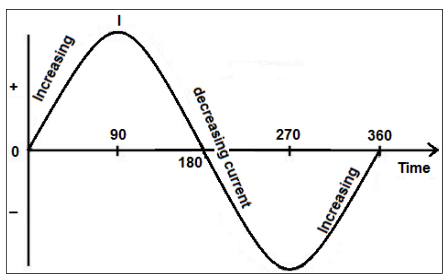



Ans. $C_T = 0.033\ \mu F$

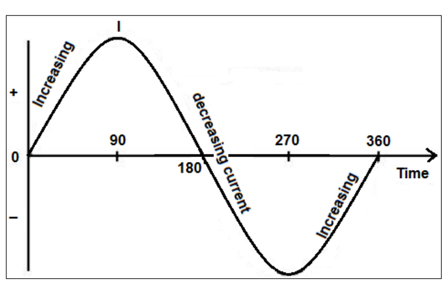
# UNIT - II
# PAPER B
# Chapter # 7
# AC FUNDAMENTALS

## 7.1 ALTERNATING CURRENT & VOLTAGE:

Alternating current abbreviation is AC, it is the flow of electric charge that periodically reverses. It starts, say, from zero, grows to a maximum value, decreases to zero, reverses in direction, reaches a maximum in the opposite direction, returns again to the original value zero, and repeats this cycle indefinitely. The AC current is denoted with “I” and measured in Amperes denoted with “A”.




AC current flows with AC voltage source. AC Voltage is the pressure provided from an electrical circuit's power source that pushes charged electrons (current) through a conducting loop, enabling them to do work such as illuminating a light. AC voltage periodically reverses. It starts, say, from zero, grows to a maximum value, decreases to zero, reverses in direction, reaches a maximum in the opposite direction, returns again to the original value zero, and repeats this cycle indefinitely. In brief, voltage = pressure, and it is denoted with “V” and measured in volts which is also denoted with “V”.



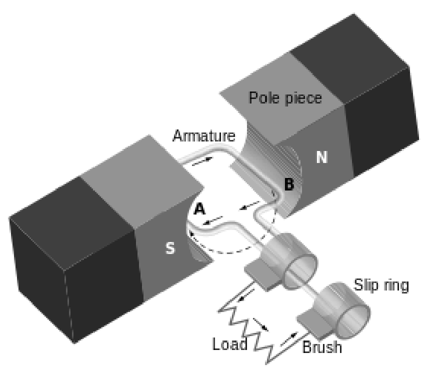
## 7.2 PRINCIPLE OF WORKING OF AC GENERATOR:

The machines which are used to generate electrical voltages are called generators. The generators which generate purely sinusoidal AC voltages are called alternators.

The basic principle of an alternator is the principle of electromagnetic induction. The sine wave is generated according to Faraday's law of electromagnetic induction. It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Such an induced e.m.f. then can be used to supply the electrical load.

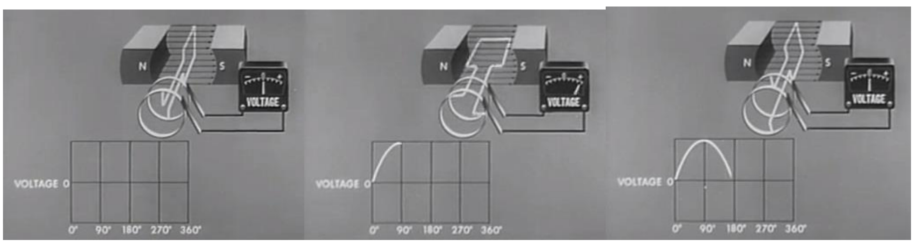
## 7.3 SIMPLE LOOP ALTERNATOR:

A basic design, called elementary generator, is to have a rectangular loop armature to cut the lines of force between the north and south poles. By cutting lines of force through rotation, it produces electrical current. The current is sent out of the generator unit through two sets of slip rings and brushes, one of which is used for each end of the armature. In this two-pole design, as the armature rotates one revolution, it generates one cycle of single phase alternating current (AC). To generate an AC output, the armature is rotated at a constant speed having the number of rotations per second to match the desired frequency (in hertz) of the AC output.

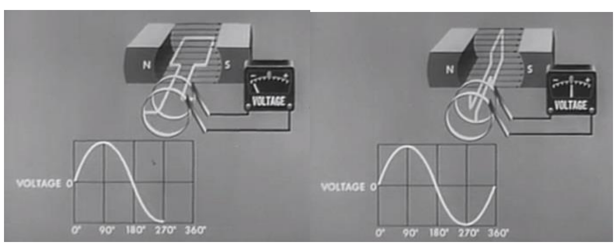



The relationship of armature rotation and the AC output can be seen in this series of pictures shown below. Due to the circular motion of the armature against the straight lines of force, a variable number of lines of force will be cut even at a constant speed of the motion. At zero degrees, the rectangular arm of the armature does not cut any lines of force, giving zero voltage output. As the armature arm rotates at a constant speed toward the 90° position, more lines are cut. The lines of force are cut at most when the armature is at the 90°

position, giving out the most current on one direction. As it turns toward the 180° position, lesser number of lines of force are cut, giving out lesser voltage until it becomes zero again at the 180° position. The voltage starts to increase again as the armature heads to the opposite pole at the 270° position. Toward this position, the current is generated on the opposite direction, giving out the maximum voltage on the opposite side. The voltage decreases again as it completes the full rotation. In one rotation, the AC output is produced with one complete cycle as represented in the sine wave.




Coil at 0º Coil at 90º Coil at 180º

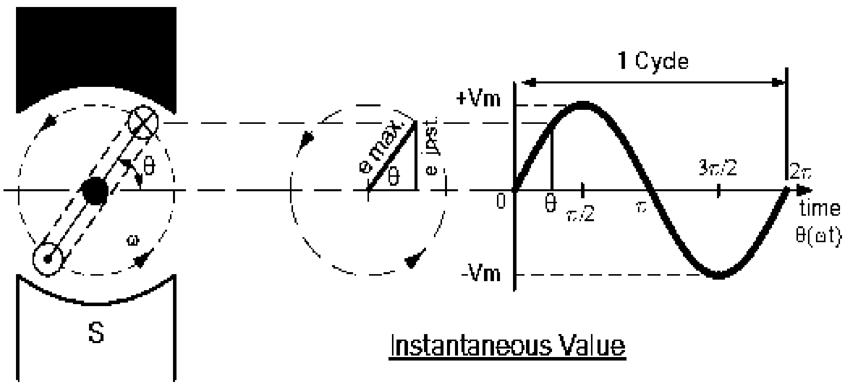


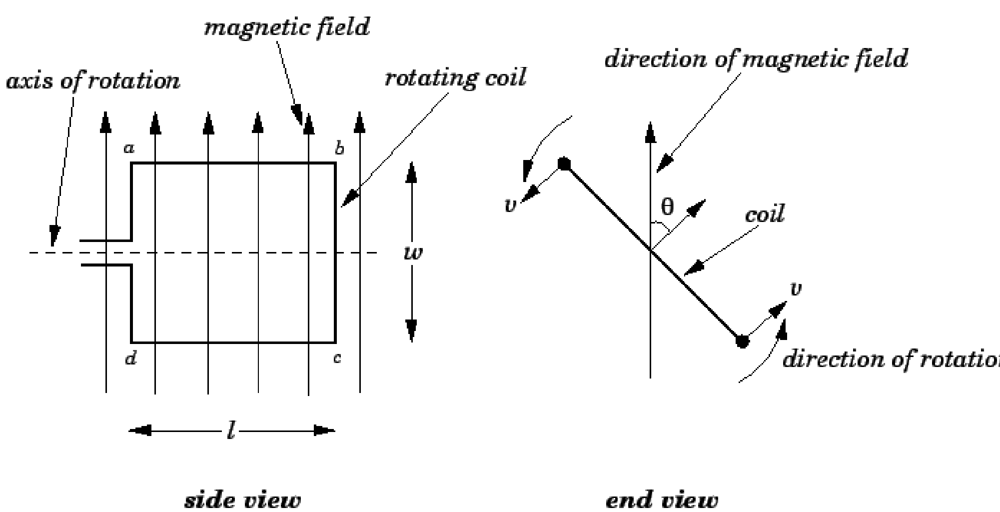

Coil at 270º Coil at 360º

## RELATION BETWEEN SPEED, POLES & FREQUENCY:

One cycle of alternating current is produced each time a pair of field poles passes over a point on the stationary winding. The relation between speed and frequency is $N = 120f/P$, where $f$ is the frequency in Hz (cycles per second), $P$ is the number of poles (2,4,6...) and $N$ is the rotational speed in revolutions per minute (RPM). The output frequency of an alternator depends on the number of poles and the rotational speed. The speed corresponding to a particular frequency is called the synchronous speed for that frequency.

## 7.4 SINUSOIDAL EMF EQUATION:




Consider a rectangular coil having N turn and rotating in a uniform magnetic field with an angular velocity of ω radians per second as shown in above figure. Let time is measured from the x-axis. Maximum flux $\Phi_m$ is linked with the coil when its plane coincides with the x-axis. In time t seconds, the coil rotates through an angle of $\theta = \omega t$, in this deflected position, the component of the flux which is perpendicular to the plane of the coil is;

$$\Phi = \Phi_m \cos \omega t.$$

Hence, flux linkage of the coil at any time are $N\Phi = N\Phi_m \cos \omega t$.

According to the Faraday's laws of electromagnetic induction the emf induced in the coil is given by the ratio of the rate of change of flux linkage of the coil. Hence, the value of instantaneous value of induced emf;

$$e = -\frac{d}{dt}(N\Phi) \quad \text{volt}$$
$$= -N.\frac{d}{dt}(\Phi_m \cos \omega t) \quad \text{volt}$$
$$= -N\Phi_m\omega(-\sin \omega t) \quad \text{volt}$$
$$= \omega N\Phi_m \sin \omega t \quad \text{volt}$$
$$= \omega N\Phi_m \sin\theta \quad \text{volt} \quad \text{---------------} \quad (i)$$

When the coil has turned through 90º i.e. when θ=90º then sin θ=1, hence e has maximum value, say $E_m$. Therefore, from the above equation (i) we get;

$$E_m = \omega N\Phi_m$$
$$= 2\pi fN\Phi_m$$

Substituting this value of $E_m$ in above equation (i), we get:

$$e = E_m \sin\omega t$$
$$= E_m \sin 2\pi f t$$

Similarly;

$$i = I_m \sin 2\pi f t$$

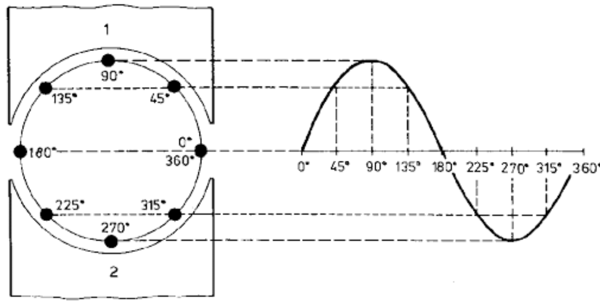
## 7.5 WAVE FORM:

A wave form is a representation of how alternating current (AC) varies with time. The most familiar AC wave form is the sine wave, which derives its name from the fact that the current or voltage varies with the sine of the elapsed time.

There are many different types of electrical wave forms available but generally they can all be broken down into two distinctive groups.

- 1. Uni-directional Wave forms – these electrical wave forms are always positive or negative in nature flowing in one forward direction only as they do not cross the zero-axis point. Common unidirectional wave forms include Square-wave timing signals, Clock pulses and Trigger pulses.
- 2. Bi-directional Wave forms – these electrical wave forms are also called alternating wave forms as they alternate from a positive direction to a negative direction constantly crossing the zero-axis point. Bi-directional wave forms go through periodic changes in amplitude, with the most common by far being the Sine-wave.

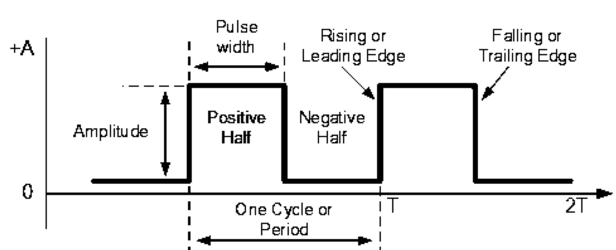
**Sine Wave Waveform** is the most common type of AC wave. The AC in most homes and offices has an oscillating voltage that produces a sine wave. When an alternator produces AC voltage, the voltage switches polarity over time, but does so in a very particular manner. When graphed over time, the “wave” traced by this voltage of alternating polarity from an alternator takes on a distinct shape, known as a sine wave as shown below.



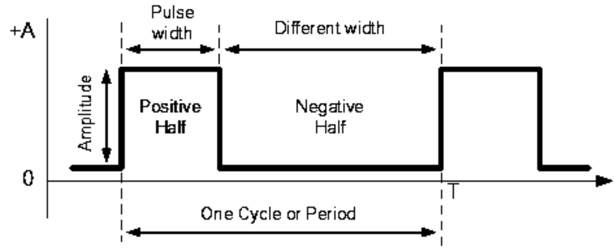

To draw this wave form, take trigonometric sine function of different angles of coil rotation in a cycle.

| Angle (°) | Sin (Angle) | Wave | Angle (°) | Sin (Angle) | Wave |
|---|---|---|---|---|---|
| 0 | 0.0000 | zero | 180 | 0.0000 | zero |
| 15 | 0.2588 | + | 195 | -0.2588 | - |
| 30 | 0.5000 | + | 210 | -0.5000 | - |
| 45 | 0.7071 | + | 225 | -0.7071 | - |
| 60 | 0.8660 | + | 240 | -0.8660 | - |
| 75 | 0.9659 | + | 255 | -0.9659 | - |
| 90 | 1.0000 | +peak | 270 | -1.0000 | -peak |
| 105 | 0.9659 | + | 285 | -0.9659 | - |
| 120 | 0.8660 | + | 300 | -0.8660 | - |
| 135 | 0.7071 | + | 315 | -0.7071 | - |
| 150 | 0.5000 | + | 330 | -0.5000 | - |
| 165 | 0.2588 | + | 345 | -0.2588 | - |
| 180 | 0.0000 | zero | 360 | 0.0000 | zero |

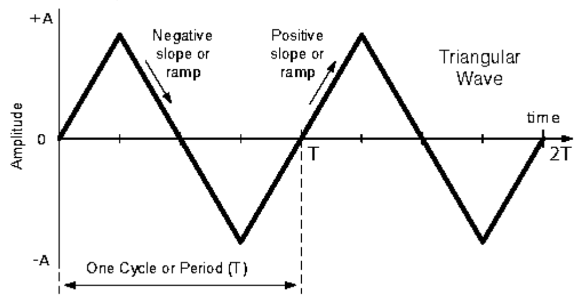
**Square-wave** are used extensively in electronic and microelectronic circuits for clock and timing control signals as they are symmetrical waveforms of equal and square duration representing each half of a cycle and nearly all digital logic circuits use square wave waveforms on their input and output gates.



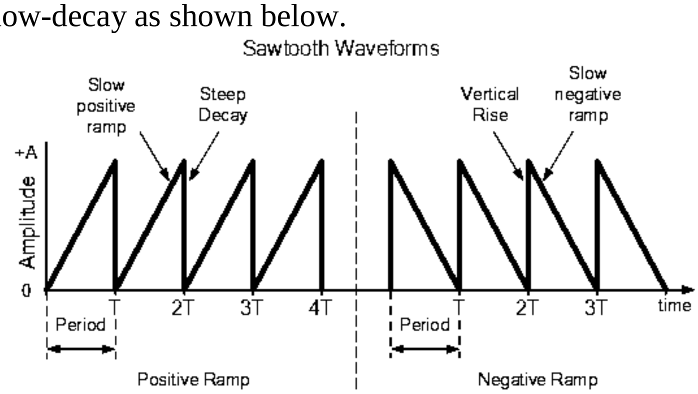
**Rectangular Waveforms** are similar to the square wave waveform above, the difference being that the two pulse widths of the waveform are of an unequal time period. Rectangular waveforms are therefore classed as "Non-symmetrical" waveforms as shown below.




**Triangular Waveforms** are generally bi-directional non-sinusoidal waveforms that oscillate between a positive and a negative peak value. Although called a triangular waveform, the triangular wave is actually more of a symmetrical linear ramp waveform because it is simply a slow rising and falling voltage signal at a constant frequency or rate. The rate at which the voltage changes between each ramp direction is equal during both halves of the cycle as shown below.



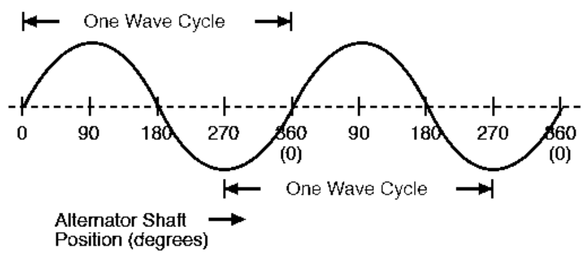

**Sawtooth Waveforms** are another type of periodic waveform. As its name suggests, the shape of the waveform resembles the teeth of a saw blade. Sawtooth waveforms can have a mirror image of themselves, by having either a slow-rising but extremely steep decay, or an extremely steep almost vertical rise and a slow-decay as shown below.



The positive ramp **Sawtooth Waveform** is the more common of the two waveform types with the ramp portion of the wave being almost perfectly linear. The Sawtooth waveform is commonly available from most function generators and consists of a fundamental frequency ( *ƒ* ) and all its integer ratios of harmonics, such as: 1/2, 1/4, 1/6 1/8 … 1/n etc. What this means in practical terms is that the **Saw-toothed Waveform** is rich in harmonics and for music synthesizers and musicians gives the quality of the sound or tonal colour to their music without any distortion.

## 7.6 CYCLE:

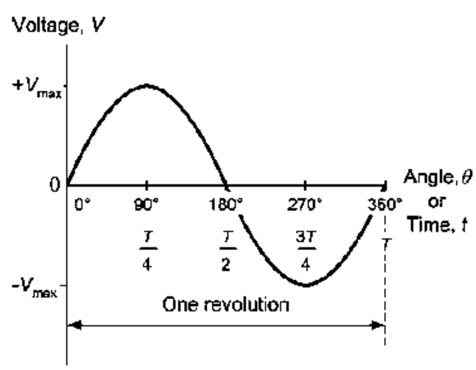
If we follow the changing voltage produced by a coil in an alternator from any point on the sine wave graph to that point when the wave shape begins to repeat itself, we would have marked exactly one cycle.




## TIME PERIOD:

The time period of a waveform is the time required for completing one full cycle. It is measured in seconds. In the figure shown the sinusoidal waveform is plotted as a function of the argument ωt, and the periodic nature of the sine wave is evident. The function repeats itself every 2π radians, and its period is therefore 2π radians. The relationship between time (T) and frequency (f) is indicated by the formulas

$T = 1/f$



## FREQUENCY:

The number of cycles per second is called frequency of the alternating quantity. It is measured in cycle per second (C/S) or Hertz (Hz). If the signal in the Figure shown above makes one complete revolution each second, the Generator produces one complete cycle of AC during each second (1 Hz). Increasing the number of revolutions to two per second will produce two complete cycles of ac per second (2 Hz).

$$f = 1 / T$$

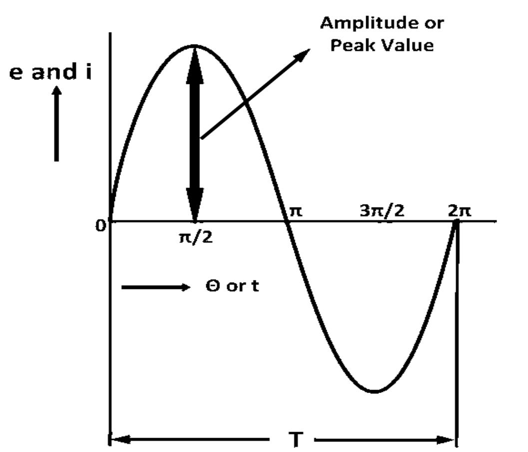
For a generator frequency can be calculated by the given formula;

$$f = PN / 120;$$

Where P is the number of magnetic poles, N is speed of rotation of generator in revolution per minute.

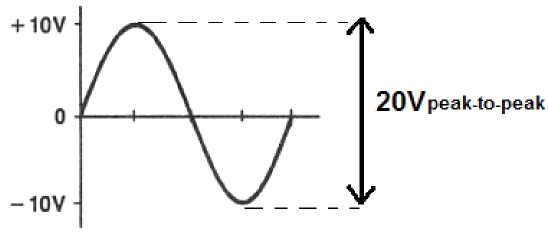
## PEAK VALUE:

One way to express the intensity, or magnitude (also called the amplitude), of an AC quantity is to measure its peak height on a waveform graph. This is known as the peak or crest value of an AC waveform: Figure below shows the peak voltage of a waveform.

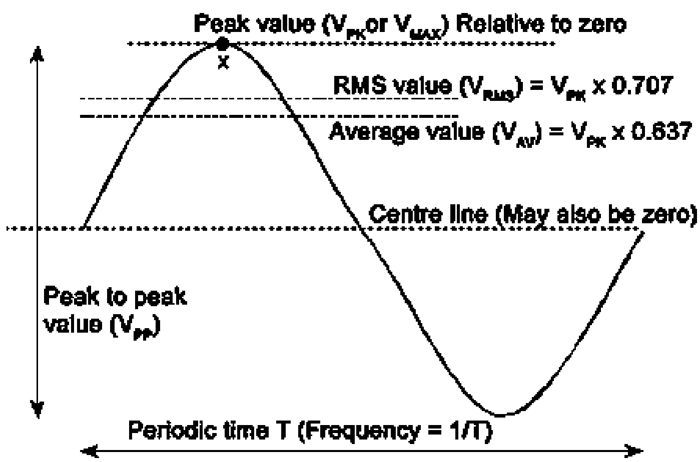



## PEAK TO PEAK VALUE:

Peak-to-peak voltage, VPP, is a voltage of waveform which is measured from the top of the waveform, called the crest, all the way down to the bottom of the waveform, called the trough. So peak-to-peak voltage is just the full vertical length of a voltage waveform from the very top to the very bottom.




## INSTANTANEOUS VALUE:




The instantaneous value of an AC signal is the value of voltage or current at one particular instant. The value may be zero if the particular instant is the time in the cycle at which the polarity of the voltage is changing. It may also be the same as the peak value, if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing. There are actually an infinite number of instantaneous values between zero and the peak value.

## AVERAGE VALUE:

The average value of an AC current or voltage is the average of all the instantaneous values during one alternation. They are actually DC values. The average value is the amount of voltage that would be indicated by a DC voltmeter if it was connected across the load resistor. Since the voltage increases from zero to peak value and decreases back to zero during one alternation, the average value must be some value between those two limits. It is possible to determine the average value by adding together a series of instantaneous values of the alternation (between 0° and 180°), and then dividing the sum by the number of instantaneous values used. The computation would show that one alternation of a sine wave has an average value equal to 0.636 times the peak value. The formula for average voltage is;

$$V_{avg} = 0.636V_{max}$$

Where $V_{avg}$ is the average voltage for one alteration, and $V_{max}$ is the maximum / peak voltage. Similarly, the formula for average current is;

$$I_{avg} = 0.636I_{max}$$

Where $I_{avg}$ is the average current for one alteration, and $I_{max}$ is the maximum /peak current.

## EFFECTIVE / RMS VALUE:

This is the value of AC signal that will have the same effect on a resistance as a comparable value of direct voltage or current will have on the same resistance. It is possible to compute the effective value of a sine wave of current to a good degree of accuracy by taking equally spaced instantaneous values of current the curve and extracting the square root of the average of the sum of the squared values. For this reason, the effective value is often called the “root mean square” (RMS) value. Therefore, $I_{eff}$ or $I_{rms}$is the average of the sum of the squares of $I_{ins}$. The effective or rms value ($I_{eff}$ or $I_{rms}$) of a sine wave of current is 0.707 times the maximum value of current (Imax).

Thus, $I_{eff}$ / $I_{rms}$ = 0.707 × $I_{max}$. Importantly, all AC voltage and current values are given as effective values.

## FORM FACTOR:

The form factor of an alternating current waveform (signal) is the ratio of the RMS (Root Mean Square) value to the average. It identifies the ratio of the direct current of equal power relative to the given alternating current.

Mathematically form factor = $\frac{I_{rms}}{I_{avg}}$

For a sine wave

Form factor = $\frac{I_{rms}}{I_{avg}} = \frac{0.707I_{max}}{0.636I_{max}} = 1.11$

## PEAK / CREST FACTOR:

Peak / Crest factor is the peak amplitude of the waveform divided by the RMS value of the wave form.

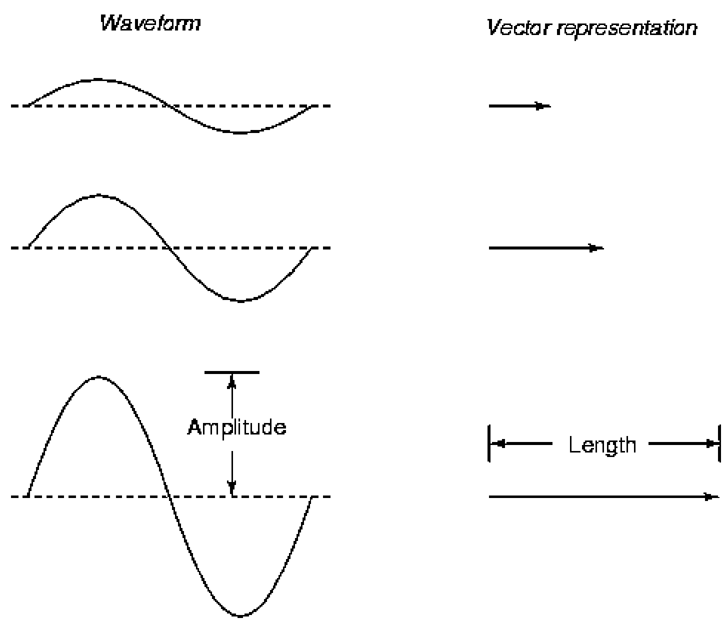
Mathematically peak factor = $\frac{I_{max}}{I_{rms}}$

For a sine wave

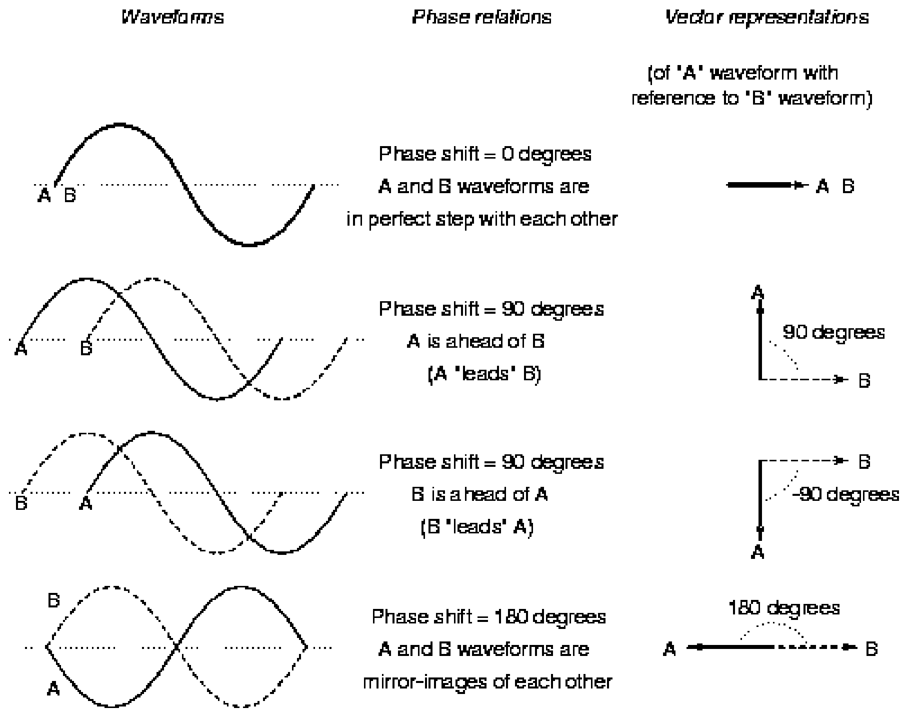
Form factor = $\frac{I_{max}}{I_{rms}} = \frac{I_{max}}{0.707I_{max}} = \frac{1}{0.707} = \sqrt{2} = 1.414$

## 7.7 REPRESENTATION OF AC QUANTITIES BY VECTOR:

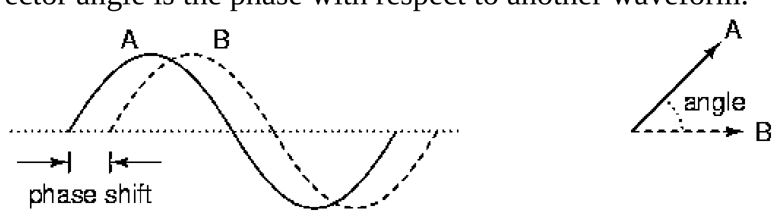
AC quantities of voltage or current can be represented in the form of a vector. The length of the vector represents the magnitude (or amplitude) of the waveform, like this: (Figure below)




Vector length represents magnitude of waveform of AC quantity, thus greater the amplitude of the waveform, the greater the length of its corresponding vector. The angle of the vector, however, represents the phase shift in degrees between the waveform in question and another waveform acting as a "reference" in time. Usually, when the phase of a waveform in a circuit is expressed, it is referenced to the power supply voltage waveform (arbitrarily stated to be "at" $0^o$). Remember that phase is always a relative measurement between two waveforms rather than an absolute property. (Figure below)




Vector angle is the phase with respect to another waveform.

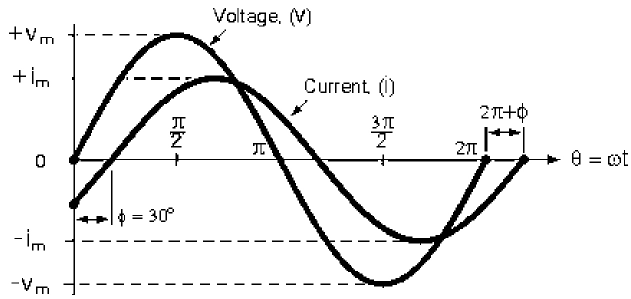



## 7.8 PHASE:

Phase represent the live wire / positive terminal where the electricity passes. Neutral is the negative side when both connected utility glows / goes live.

## PHASE DIFFERENCE:

Sometimes when we are analyzing alternating waveforms we may need to know the position of the Phasor, representing the Alternating Quantity at some particular instant in time especially when we want to compare two different waveforms on the same axis, for example voltage and current. We have assumed in the waveform above that the waveform starts at time t = 0 with a corresponding phase angle in either degrees or radians. But if a second waveform starts to the left or to the right of this zero-point or we want to represent in Phasor notation the relationship between the two waveforms then we will need to take into account this phase difference, Φ of the waveform. Consider the diagram below.




The generalized mathematical expression to define these two sinusoidal quantities will be written as:

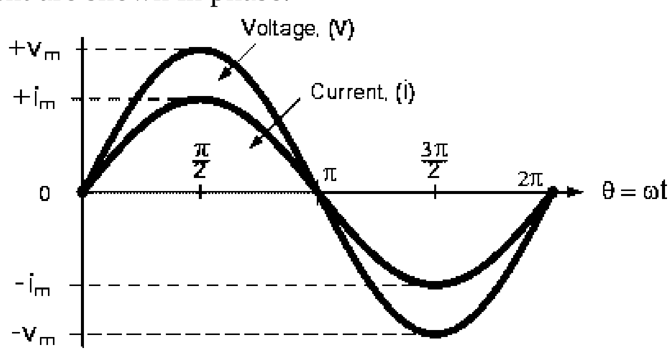
$$v_{(t)} = V_m \sin(\omega t)$$

$$i_{(t)} = I_m \sin(\omega t - \phi)$$

The current i, is lagging the voltage v, by angle Φ and in our example above this is $30^o$. So, the difference between the two Phasor representing the two sinusoidal quantities is angle Φ and the resulting Phasor diagram will be.

## IN PHASE:

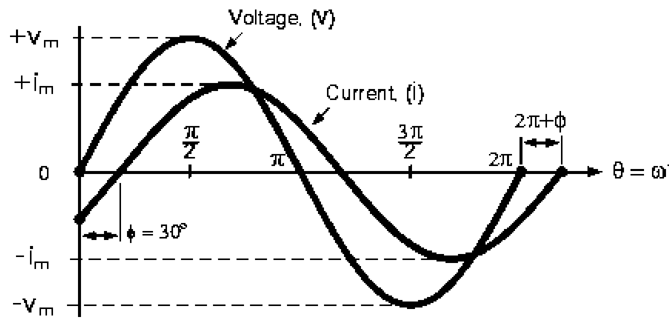
The phase involves the relationship between the position of the amplitude crests and troughs of two waveforms. Phase can be measured in distance, time, or degrees. If the peaks of two signals with the same frequency are in exact alignment at the same time, they are said to be in phase. In figure voltage and current are shown in phase.




## OUT OF PHASE:

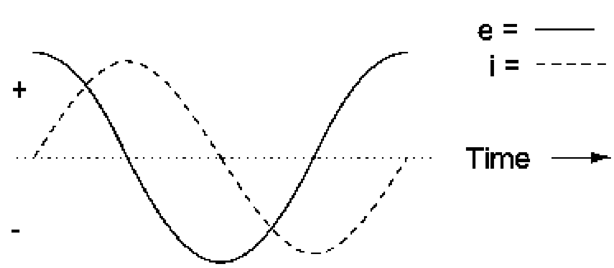
The phase involves the relationship between the position of the amplitude crests and troughs of two waveforms. Phase can be measured in

distance, time, or degrees. If the peaks of two signals with the same frequency are not in exact alignment at the same time, they are said to be out of phase.
Now let's consider that the voltage, v and the current, i have a phase difference between themselves of $30^o$, so ($\Phi = 30^o$ or $\pi/6$ radians). As both alternating quantities rotate at the same speed, i.e. they have the same frequency, this phase difference will remain constant for all instants in time, then the phase difference of $30^o$ between the two quantities is represented by phi, $\Phi$ as shown below.



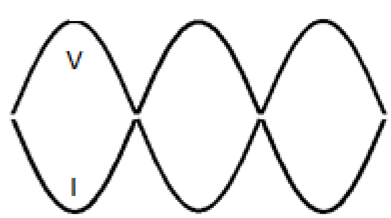

## PHASE QUADRATURE:

If the peaks of two waves with the same frequency are 90 degree (Quarter of wave cycle) out of phase, they are said to be in phase quadrature.




## ANTI PHASE:

If the peaks of two waves with the same frequency are 180 degree (Half wave cycle) out of phase, they are said to be in anti-phase.



## LAGGING & LEADING:

When two waveforms are out of phase, then the way to express the time difference between the two is by stating the angle difference for one cycle, i.e., the angle value of the first waveform when the other one has a zero value. This is shown in Figure, where there is a phase difference of 30° between the waveforms A and B.

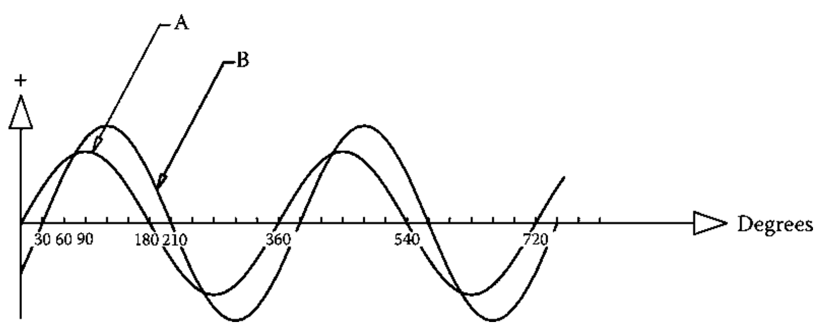
In conjunction with the phase difference are two other terms called leading and lagging.

When the waveform A is ahead of B (i.e., when A reaches its maximum value before B reaches its maximum value), it is said to be leading waveform B. At the same time, B is behind (following) A, and it is said to be lagging A.

Note that when A is leading B, it also reaches its minimum value and zero value before B reaches those values. The amount of leading or lagging is expressed by the phase angle, which is the phase difference between the two waveforms.

To compare the phase between two sine wave, we should follow the conditions below,

- The signal should be in sinusoidal nature.
- The two signals should have the same frequency
- Both the signal must be written in positive amplitude
- The phase angle must always be less than 180°.




## 7.9 DRAW PHASOR DIAGRAM:

Phasor diagrams are a graphical way of representing the magnitude and directional relationship between two or more alternating quantities. Sinusoidal waveforms of the same frequency can have a Phase Difference between themselves which represents the angular difference of the two sinusoidal waveforms. Also the terms "lead" and "lag" as well as "in-phase" and "out-of-phase" are commonly used to indicate the relationship of one waveform to the other with the generalized sinusoidal expression given as: $A_{(t)} = A_m \sin(\omega t \pm \Phi)$ representing the sinusoid in the time-domain form. But when presented mathematically in this way it is sometimes difficult to visualize this angular or phasor difference between two or more sinusoidal waveforms. One way to overcome this problem is to represent the sinusoids graphically within the special or phasor-domain form by using Phasor Diagrams, and this is achieved by the rotating vector method.

Basically, a rotating vector, simply called a "Phasor" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time.

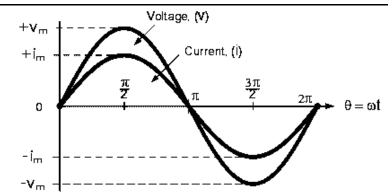
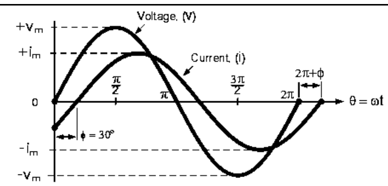
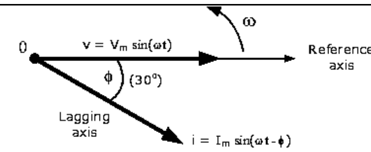
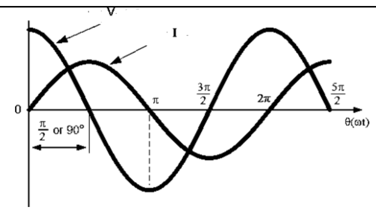
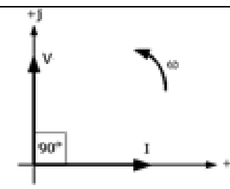
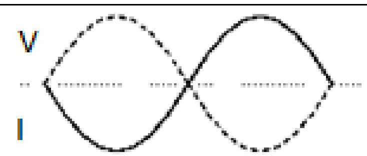
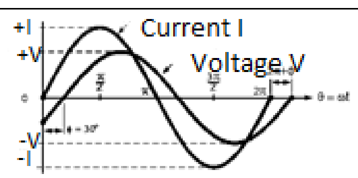
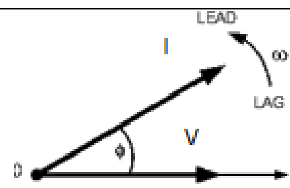
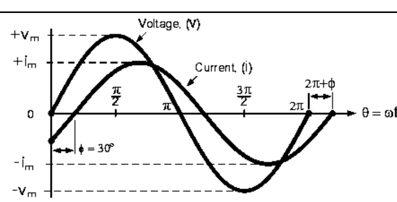
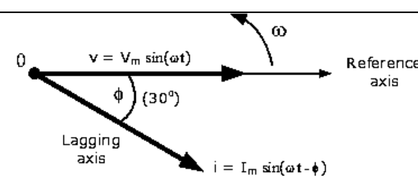
A phasor is a vector that has an arrow head at one end which signifies partly the maximum value of the vector quantity (V or I) and partly the end of the vector that rotates.

Generally, vectors are assumed to pivot at one end around a fixed zero point known as the "point of origin" while the arrowed end representing the quantity, freely rotates in an anti-clockwise direction at an angular velocity, ($\omega$) of one full revolution for every cycle. This anti-clockwise rotation of the vector is considered to be a positive rotation. Likewise, a clockwise rotation is considered to be a negative rotation.

Although the both the terms vectors and phasors are used to describe a rotating line that itself has both magnitude and direction, the main difference between the two is that a vectors magnitude is the "peak value" of the sinusoid while a phasors magnitude is the "rms value" of the sinusoid. In both cases the phase angle and direction remain the same.

The phase of an alternating quantity at any instant in time can be represented by a phasor diagram, so phasor diagrams can be thought of as "functions of time". A complete sine wave can be constructed by a single vector rotating at an angular velocity of $\omega = 2\pi f$, where $f$ is the frequency of the waveform. Then a Phasor is a quantity that has both "Magnitude" and "Direction".

Generally, when constructing a phasor diagram, angular velocity of a sine wave is always assumed to be: $\omega$ in rad/sec.

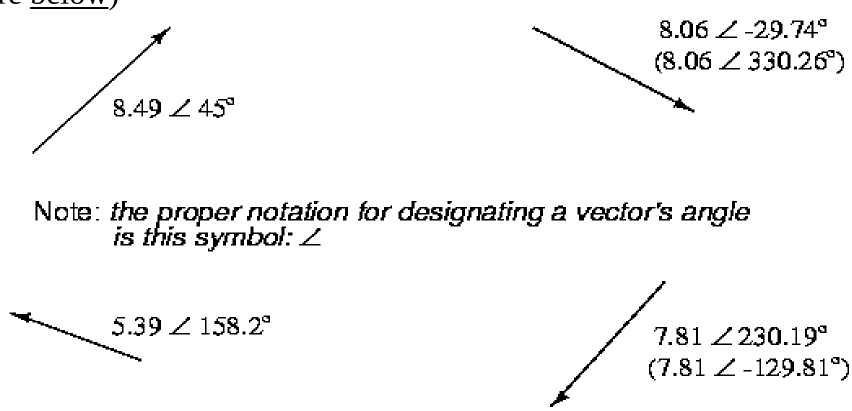
| Position of Quantities | Wave Form | Phasor Diagram |
|---|---|---|
| Voltage and Current are in phase |  |  |
| Voltage and Current are out of phase |  |  |
| Voltage and Current are in phase quadrature |  |  |
| Voltage and Current are in anti-phase |  |  |
| Current is Leading Voltage |  |  |
| Current is Lagging Voltage |  |  |

## 7.10 <u>COMPLEX NUMBERS:</u>

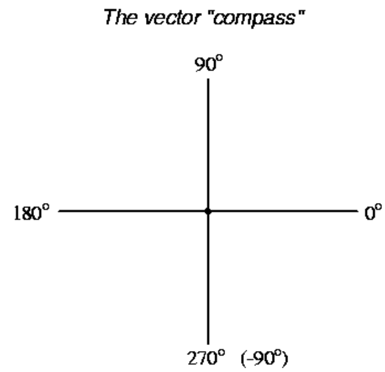
Complex numbers are that which consists two parts; a real number and an imaginary number. The standard format for complex number is a + bj with the real number first (a) and the imaginary number last (b). Technically any real number or imaginary number can be considered as complex number. Complex does not mean complicated; it means that the two types of numbers combine to form a complex. Real numbers are tangible values that can be plotted on horizontal number line, such as fractions, integers or any countable number that you can think of. Imaginary numbers are abstract concepts that are used when you need the square root of a negative number. Two forms of complex numbers which are used frequently in calculation to solve numerical problems by an electrical technologist or engineer are Polar and Rectangular forms.

## <u>POLAR FORM OF AC QUANTITY:</u>

Polar form is where a complex number is denoted by the length (otherwise known as the magnitude, absolute value, or modulus) and the angle of its vector (usually denoted by an angle symbol that looks like this: ∠). Here are two examples of vectors and their polar notations: (Figure <u>below</u>)



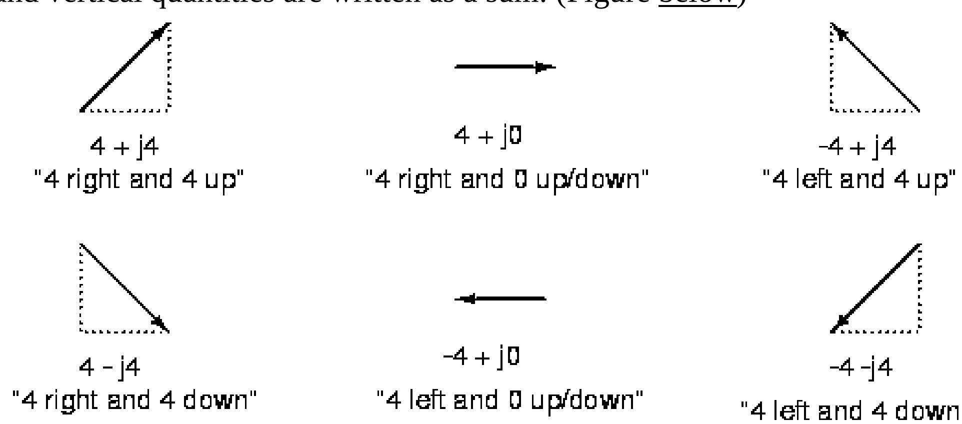

Standard orientation for vector angles in AC circuit calculations defines $0^o$ as being to the right (horizontal), making $90^o$ straight up, $180^o$ to the left, and $270^o$ straight down. Please note that vectors angled "down" can have angles represented in polar form as positive numbers in excess of 180, or negative numbers less than 180. For example, a vector angled ∠ $270^o$ (straight down) can also be said to have an angle of $-90^o$. (Figure <u>below</u>) The above vector on the right (7.81 ∠ $230.19^o$) can also be denoted as 7.81 ∠ $-129.81^o$.




## RECTANGULAR FORM OF AC QUANTITY:

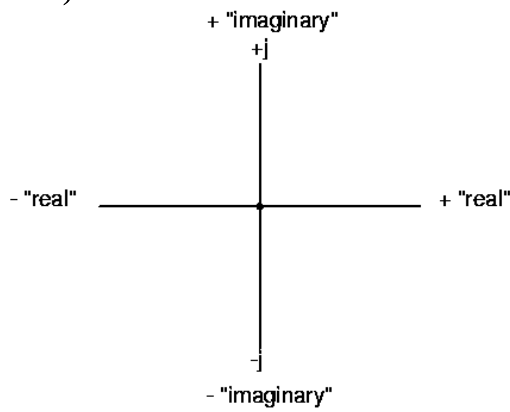
Rectangular form, on the other hand, is where a complex number is denoted by its respective horizontal and vertical components. In essence, the angled vector is taken to be the hypotenuse of a right triangle, described by the lengths of the adjacent and opposite sides. Rather than describing a vector's length and direction by denoting magnitude and angle, it is described in terms of "how far left/right" and "how far up/down."

These two-dimensional figures (horizontal and vertical) are symbolized by two numerical figures. In order to distinguish the horizontal and vertical dimensions from each other, the vertical is prefixed with a lower-case "i" (in pure mathematics) or "j" (in electronics). These lower-case letters do not represent a physical variable (such as instantaneous current, also symbolized by a lower-case letter "i"), but rather are mathematical operators used to distinguish the vector's vertical component from its horizontal component. As a complete complex number, the horizontal and vertical quantities are written as a sum: (Figure below)




In "rectangular" form the vector's length and direction are denoted in terms of its horizontal and vertical span, the first number representing the horizontal ("real") and the second number (with the "j" prefix) representing the vertical ("imaginary") dimensions.

The horizontal component is referred to as the real component, since that dimension is compatible with normal, scalar ("real") numbers. The vertical component is referred to as the imaginary component, since that dimension lies in a different direction, totally alien to the scale of the real numbers. (Figure below)

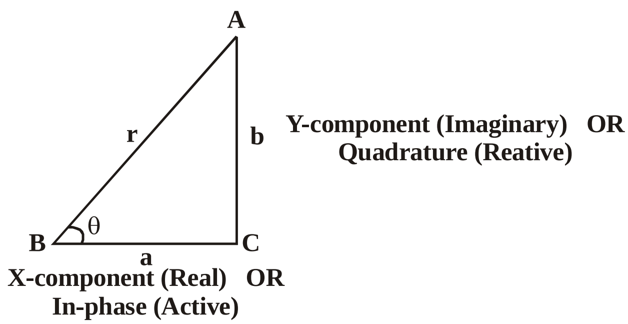



The "real" axis of the graph corresponds to the familiar number line we saw earlier: the one with both positive and negative values on it. The "imaginary" axis of the graph corresponds to another number line situated at $90^o$ to the "real" one. Vectors being two-dimensional things, we must have a two-dimensional "map" upon which to express them, thus the two number lines perpendicular to each other.

Either method of notation is valid for complex numbers. The primary reason for having two methods of notation is for ease of longhand calculation, rectangular form lending itself to addition and subtraction, and polar form lending itself to multiplication and division.

## 7.11 CONVERSIONS FROM R-P AND P-R FORM:

Conversion between the two notational forms involves simple trigonometry. To convert from polar to rectangular, find the real component by multiplying the polar magnitude by the cosine of the angle, and the imaginary component by multiplying the polar magnitude by the sine of the angle. This may be understood more readily by drawing the quantities as sides of a right triangle, the hypotenuse of the triangle representing the vector itself (its length and angle with respect to the horizontal constituting the polar form), the horizontal and vertical sides representing the "real" and "imaginary" rectangular components, respectively: (Figure below)




To convert from rectangular to polar, find the polar magnitude through the use of the Pythagorean Theorem (the polar magnitude is the hypotenuse of a right triangle, and the real and imaginary components are the adjacent and opposite sides, respectively), and the angle by taking the arctangent of the imaginary component divided by the real component:

## EXAMPLE 1:

Convert $5 \angle 36.87^{\circ}$ into rectangular form?

**SOLUTION:**

| | |
|---|---|
| $5 \angle 36.87^{\circ}$ | (Polar form) |
| $(5)(\cos 36.87^{\circ}) = 4$ | (real component) |
| $(5)(\sin 36.87^{\circ}) = 3$ | (imaginary component) |
| $4 + j3$ | (rectangular form) |

## EXAMPLE 2:

Convert $-5\angle 30^{\circ}$ into rectangular form?

**SOLUTION:**

$-5 \cos 30^{\circ} = -4.33$ Real component

$-5 \sin 30^{\circ} = -2.5$ Imaginary component

$-4.33 - j2.5$ (Rectangular form)

## EXAMPLE 3:

Convert $15 \angle -60^{\circ}$ into rectangular form?

**SOLUTION:**

$15 \cos(-60^{\circ}) = 7.5$ Real component

$15 \sin(-60^{\circ}) = -13$ Imaginary component

$7.5 - j13$ (Rectangular form)

## EXAMPLE 4:

Convert $4 + j3$ into polar form?

**SOLUTION:**

$Z = 4 + j3$ ----------- Rectangular form

$r = \sqrt{a^2 + b^2} = \sqrt{4^2 + 3^2} = 5$

$\tan\theta = b / a$

$$\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{3}{4} = 36.87°$$

$$Z = r\angle\theta = 5 \angle 36.87° \text{ (Polar form)}$$

## EXAMPLE 5:

Convert 8 - j6 into polar form?

## SOLUTION:

8 - j6 Rectangular form

$C=\sqrt{a^2+b^2}$ Pythagorean Theorem

Polar Magnitude = $\sqrt{8^2+(-6)^2}$ = 10

Polar angle = $\tan^{-1}\left(\frac{-6}{8}\right)$ = -36.87º

Polar form = 10 < - 36.87º

# EXERCISE # 07

## PART-A

**Chose the correct answer**

1. **Current (or voltage) is said to be alternating when it changes in;**
   (a) Magnitude only (b) Direction only
   (c) Both a and b (d) None of these
2. **The working principle of AC generator is;**
   (a) Electrostatics (b) Magnetic locking
   (c) Electro chemical (d) Electromagnetic induction
3. **Basic single loop generator consists on:**
   (a) 4 poles (b) 2 poles (c) 1 pole (d) 6 poles
4. **Single loop generator consists of --------- slipring.**
   (a) 1 (b) 2 (c) 4 (d) None
5. **Frequency in Pakistan is:**
   (a) 25 c/s (b) 40 c/s (c) 50 c/s (d) 60 c/s
6. **The sign wave which complete 50 cycles per second, its frequency will be**
   (a) 25c/s (b) 50c/s (c) 75c/s (d) 100c/s
7. **One complete set of positive and negative values of an alternating quantity is known as;**
   (a) Waveform (b) Cycle (c) Amplitude (d) Phasor
8. **In single loop generator when coil is parallel to lines of force, then induced emf.**
   (a) Zero (b) Low (c) High (d) Maximum
9. **Number of cycles per second in any conductor is called**
   (a) Frequency (b) 1 cycle (c) Time period (d) 1 sec
10. **The unit of frequency is**
    (a) Cycle (b) Second (c) Cycle/sec (d) Sec/cycle
11. **Formula to find the frequency of alternator is:**
    (a) PN/60 (b) PN/120 (c) 120/PN (d) 120P/N
12. **The value of an alternating current at any given instant is:**
    (a) Maximum value (b) Peak value
    (c) Instantaneous value (d) R.M.S. value
13. **R.M.S. value of the current in sine wave is:**
    (a) 0.707Im (b) 7.07Im (c) 0.636Im (d) 6.36Im
14. **Average value of the current in sine wave is:**
    (a) 6.36Im (b) 0.636Im (c) 7.07Im (d) 0.707Im

**15. The time period of a sine wave of frequency of 50 c/s is:**
(a) 5 sec (b) 0.02 sec (c) 25 sec (d) 50 sec

**16. Sine wave equation of current can be written as:**
(a) i = Im sin $\phi$ (b) i = Im sin $\omega$t (c) I = Im sin $2\pi$ft (d) All

**17. The ratio of R.M.S value and average value is called:**
(a) Peak factor (b) Form factor (c) Crest factor (d) Amplitude

**18. Form factor =**
(a) Average value / RMS. value (b) R.M.S value / average value
(c) Max. value / R.M.S value (d) Max. value / Average value

**19. The ratio between R.M.S value and Average value is called:**
(a) Peak factor (b) Form factor
(c) Crest factor (d) Amplitude factor

**20. The Value of Form Factor is:**
(a) 1 (b) 1.11 (c) 1.414 (d) 0.707

**21. Peak Factor =**
(a) Average value / RMS. value (b) R.M.S value / average value
(c) Max. value / R.M.S value (d) Max. value / Average value

**22. Ratio between maximum value and R.M.S value of A.C Wave is called:**
(a) Peak factor (b) Amplitude factor
(c) Crest factor (d) All

**23. The Value of Peak to Peak factor is:**
(a) 1 (b) 1.11 (c) 1.414 (d) 0.707

**24. The Maximum Value of sine wave is 10A, The R.M.S value will be:**
(a) 7.07 (b) 10 (c) 14.14 (d) 28.28

**25. A 50c/s frequency sine wave has time period:**
(a) 1/50sec (b) 5sec (c) 25sec (d) 50sec

**26. If peak value of sine wave is 10A then its average value will be:**
(a) 0.636 (b) 0.707 (c) 6.36 (d) 7.07

**27. When two AC waves having the same frequency start & finish from same point collectively then these are called:**
(a) Phase (b) Out of phase (c) In phase (d) None

**28. The line which has is specific direction and length is called:**
(a) Scalar (b) Vector (c) Phaser (d) Both b & c

**29. The value of j =**
(a) $\sqrt{-1}$ (b) $\sqrt{1}$ (c) -1 (d) None

**30. The waveform which is behind the other waveform w.r.t. time or angle is called;**

(a) Lagging wave (b) Leading wave
(c) Anti phase wave (d) Phase quadrature wave

**31. The waveform which is ahead the other waveform w.r.t. time or angle is called;**

(a) Lagging wave (b) Leading wave
(c) Anti phase wave (d) Phase quadrature wave

**32. This is an example of rectangular form.**

(a) $3 + j4$ (b) $5 < 30°$
(c) 5(Cos30 + Sin30) (d) All of these

**33. This is an example of polar form.**

(a) $3 + j4$ (b) $5 < 30°$
(c) 5(Cos30 + Sin30) (d) All of these

## ANSWERS KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | b | 3. | b | 4. | b | 5. | c |
| 6. | b | 7. | b | 8. | a | 9. | a | 10. | c |
| 11. | b | 12. | c | 13. | a | 14. | b | 15. | b |
| 16. | d | 17. | b | 18. | b | 19. | b | 20. | b |
| 21. | c | 22. | a | 23. | c | 24. | a | 25. | a |
| 26. | a | 27. | c | 28. | c | 29. | a | 30. | a |
| 31. | b | 32. | a | 33. | b | | | | |

# PART-B

**Give the short answer of the following questions**

1. What is alternating current?
2. State the working principles of Ac generator.
3. The speed of 2 pole generator is 3000 r.p.m. Find its frequency.
4. Write the equations of sinusoidal e.m.f. & current.
5. Write the names of 4 types of waveforms.
6. Define Square Wave.
7. What is saw tooth wave?
8. What is sine wave?
9. Define cycle.
10. What is time period?
11. Define frequency.
12. Define Instantaneous Value.
13. Define maximum value of A.C.
14. Define Average value of A.C.
15. What do you mean by RMS value?

16. Define the from factor.
17. Define Peak factor, write its formula too.
18. What is phase and phase angle?
19. What is difference between in phase & out of phase.
20. Define Lag and Lead in A.C. Value.
21. Define Phase difference.
22. Convert 3+j4 into polar form.
23. Convert 30∠30 into rectangular form.
24. Add 8 + j7 and -5 - j3

# PART-C

**Give the detailed answer of the following questions.**

1) Explain the construction and working principle of simple single loop AC Generator.
2) What is relation between speed of A.C Generator, Number of poles and frequency, explained.
3) Write down the method for drawing the Sine wave.
4) Derive the equation of sine wave.
5) Define Wave form, state its types briefly.
6) Define lagging and leading values of A.C. explain it with wave form and vector diagrams.
7) Define polar form and rectangular form, how can we convert them, explain.

# PART-D

**SOLVE THE PROBLEMS**

Q.1: Convert $V = 5 + j12$ into polar form.

Ans. $13 \angle 67.38^\circ$

Q.2: $Z = 4 + j5$ Convert it into polar form

Ans. $6.4 \angle 51.34^\circ$

Q.3: $Z = 35 + j31.42$ Convert it into polar form.

Ans. $47.03 \angle 41.9^\circ$

Q.4: Convert $14.28 \angle 45.57^\circ$ into (Rectangular) form.

Ans. $10 + j\,10.2$

Q.5: Convert $29.749 \angle -32.82^\circ$ into vector form.

Ans. $25 – j16.124$

Q.6: Convert $10 \angle 53.13^\circ$ into Vector form.

Ans. $6 + j8$

Q.7: Add "$40 + j20$" to $20 + j120$ and convert into polar form.

Ans. $152.31 \angle 66.8^\circ$

Q.8: Subtract '$10 + j30$' from $20 - j20$ and convert into polar form.

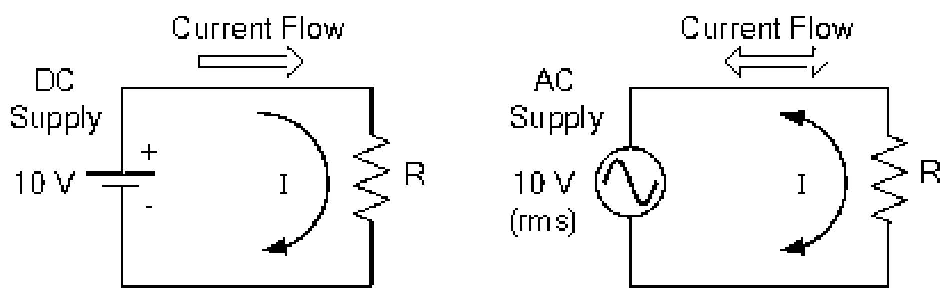
Ans. $50.99 \angle 78.69^\circ$

# Chapter # 8
# AC CIRCUITS
# (SINGLE PHASE)

## 8.1 AC THROUGH PURE RESISTIVE LOADS:

When using pure resistors in AC circuits that have negligible values of inductance or capacitance, the same principals of Ohm's Law, circuit rules for voltage, current and power (and even Kirchhoff's Laws) apply as they do for DC resistive circuits the only difference this time is in the use of the instantaneous "peak-to-peak" or "rms" quantities. When working with AC alternating voltages and currents it is usual to use only "rms" values to avoid confusion. Also, the schematic symbol used for defining an AC voltage source is that of a "wave" line as opposed to a battery symbol for DC and this is shown below.

## SYMBOL REPRESENTATION OF DC AND AC SUPPLIES

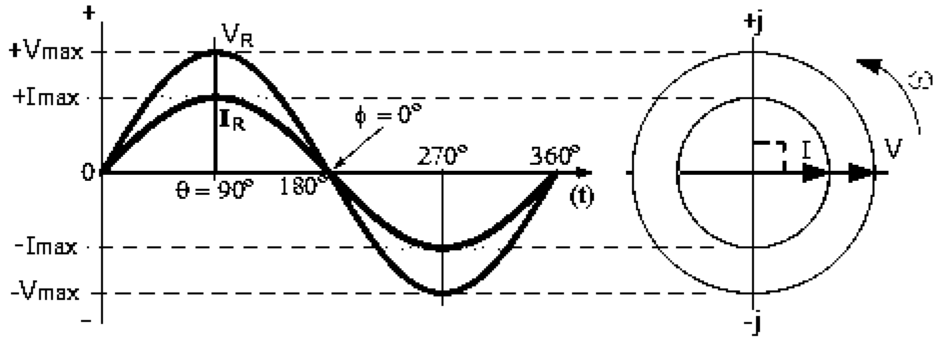



Resistors are "passive" devices that are they do not produce or consume any electrical energy, but convert electrical energy into heat. In DC circuits the linear ratio of voltage to current in a resistor is called its resistance. However, in AC circuits this ratio of voltage to current depends upon the frequency and phase difference or phase angle (φ) of the supply. So when using resistors in AC circuits the term Impedance, symbol **Z** is the generally used and we can say that DC resistance = AC impedance, R = Z.

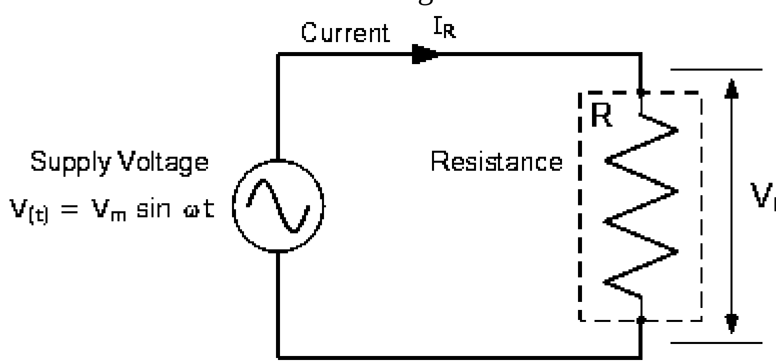
It is important to note, that when used in AC circuits, a resistor will always have the same resistive value no matter what the supply frequency from DC to very high frequencies, unlike capacitor and inductors.

For resistors in AC circuits the direction of the current flowing through them has no effect on the behavior of the resistor so will rise and fall as the voltage rises and falls. The current and voltage reach maximum, fall through zero and reach minimum at exactly the same time. i.e., they rise and fall simultaneously and are said to be "in-phase" as shown below.

## PHASE RELATIONSHIP AND VECTOR DIAGRAM




We can see that at any point along the horizontal axis that the instantaneous voltage and current are in-phase because the current and the voltage reach their maximum values at the same time that is their phase angle θ is $0^o$. Then these instantaneous values of voltage and current can be compared to give the Ohmic value of the resistance simply by using ohms law. Consider below the circuit consisting of an AC source and a resistor.




The instantaneous voltage across the resistor, $V_R$ is equal to the supply voltage, $V_t$ and is given as:

$$V_R = V_{max} \sin\omega t$$

The instantaneous current flowing in the resistor will therefore be:

$$I_R = \frac{V_R}{R} = \frac{V_{max}}{R}\sin\omega t = I_{max}\sin\omega t$$

As the voltage across a resistor is given as $V_R$ = I.R, the instantaneous voltage across the resistor above can also be given as:

$$V_R = I_{max} R \sin\omega t$$

In purely resistive series AC circuits, all the voltage drops across the resistors can be added together to find the total circuit voltage as all the voltages are in-phase with each other. Likewise, in a purely resistive parallel AC circuit, all the individual branch currents can be added together to find the total circuit current because all the branch currents are in-phase with each other.

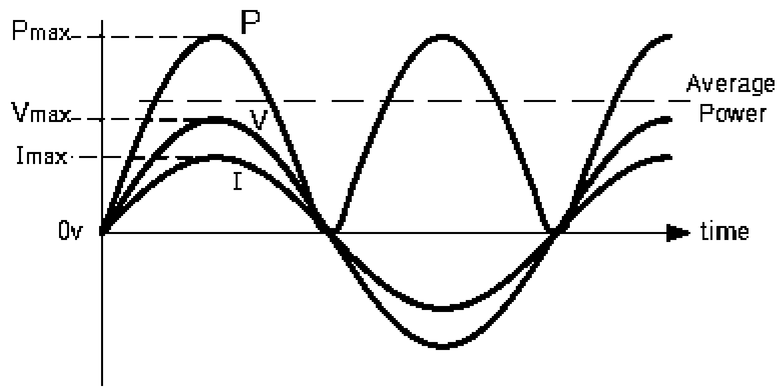
Since for resistors in AC circuits the phase angle φ between the voltage and the current is zero, then the power factor of the circuit is given as cos $\Phi^o = 1.0$. The power in the circuit at any instant in time can be found by multiplying the voltage and current at that instant.

Then the power (P), consumed by the circuit is given as;

$P = V_{rms} I \cos \Phi$ in watts. But since $\cos \Phi = 1$ in a purely resistive circuit, the power consumed is simply given as, $P = V_{rms} I$ the same as for Ohm's Law.

This then gives us the "Power" waveform and which is shown below as a series of positive pulses because when the voltage and current are both in their positive half of the cycle the resultant power is positive. When the voltage and current are both negative, the product of the two negative values gives a positive power pulse.

## POWER WAVEFORM IN A PURE RESISTANCE




Then the power dissipated in a purely resistive load fed from an AC rms supply is the same as that for a resistor connected to a DC supply and is given as:

$$P = V_{R(rms)} \times I_{rms} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

- Where:
- P is the average power in Watts
- $V_{rms}$ is the rms supply voltage in Volts
- $I_{rms}$ is the rms supply current in Amps
- R is the resistance of the resistor in Ohm's (Ω) – should really be Z to indicate impedance.

The heating effect produced by an AC current with a maximum value of Imax is not the same as that of a DC current of the same value. To compare the AC heating effect to an equivalent DC the rms values must be used. Any resistive heating element such as Electric Fires, Toasters, Kettles, Irons, and Water Heaters etc. can be classed as a resistive AC circuit and we use resistors in AC circuits to heat our homes and water.

Then to summarize, in a pure Ohmic AC Resistance, the current and voltage are both said to be “in-phase” as there is no phase difference between them. The current flowing through the resistor is directly proportional to the voltage across it with this linear relationship in an AC circuit being called Impedance. As with DC circuits, Ohm’s Law can be used when working with resistors in AC circuits to calculate the resistors voltages, currents and power.

## RESISTORS IN AC CIRCUITS

## EXAMPLE NO.1

A 1000W heating element is connected to a 250v AC supply voltage. Calculate the impedance (AC resistance) of the element when it is hot and the amount of current taken from the supply.

$$\text{Current}, I = \frac{P}{V} = \frac{1000W}{250V} = 4\ \text{amps}$$

$$Z = \frac{V}{I} = \frac{250}{4} = 62.5\Omega$$

## RESISTORS IN AC CIRCUITS

## EXAMPLE NO.2

Calculate the power being consumed by a 100Ω resistive element connected across a 240v supply. As there is only one component connected to the supply, the resistor, then $V_R = V_S$

$$\text{Current}, I = \frac{V_R}{R} = \frac{240}{100} = 2.4\ \text{amps}$$

$$\text{Power consumed}, P = I^2R = 2.4^2 \times 100 = 576W$$

## PURE INDUCTIVE AC CIRCUIT:

An alternating current circuit which contains an AC Inductance, the flow of current through an inductor behaves very differently to that of a steady state DC voltage. Now in an AC circuit, the opposition to the current flowing through the coils windings not only depends upon the inductance of the coil but also the frequency of the applied voltage waveform as it varies from its positive to negative values.

The actual opposition to the current flowing through a coil in an AC circuit is determined by the AC Resistance of the coil with this AC resistance being represented by a complex number. But to distinguish a DC resistance value from an AC resistance value, which is also known as Impedance, the term Reactance is used.

Like resistance, reactance is measured in Ohm's but is given the symbol X to distinguish it from a purely resistive R value and as the component in question is an inductor, the reactance of an inductor is called Inductive Reactance, ($X_L$) and is measured in Ohms. Its value can be found from the formula.

## INDUCTIVE REACTANCE:

$$X_L = 2\pi f L$$

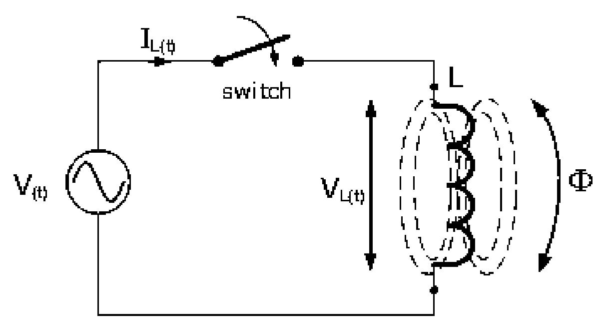
Where: $X_L$ is the Inductive Reactance in Ohms, $f$ is the frequency in Hertz and L is the inductance of the coil in Henry.

We can also define inductive reactance in radians, where Omega, ω equals $2\pi f$.

$$X_L = \omega L$$

So, whenever a sinusoidal voltage is applied to an inductive coil, the back emf opposes the rise and fall of the current flowing through the coil and in a purely inductive coil which has zero resistance or losses, this impedance (which can be a complex number) is equal to its inductive reactance. Also, reactance is represented by a vector as it has both a magnitude and a direction (angle).

## AC INDUCTANCE WITH A SINUSOIDAL SUPPLY




This simple circuit above consists of a pure inductance of L Henry (H), connected across a sinusoidal voltage given by the expression:

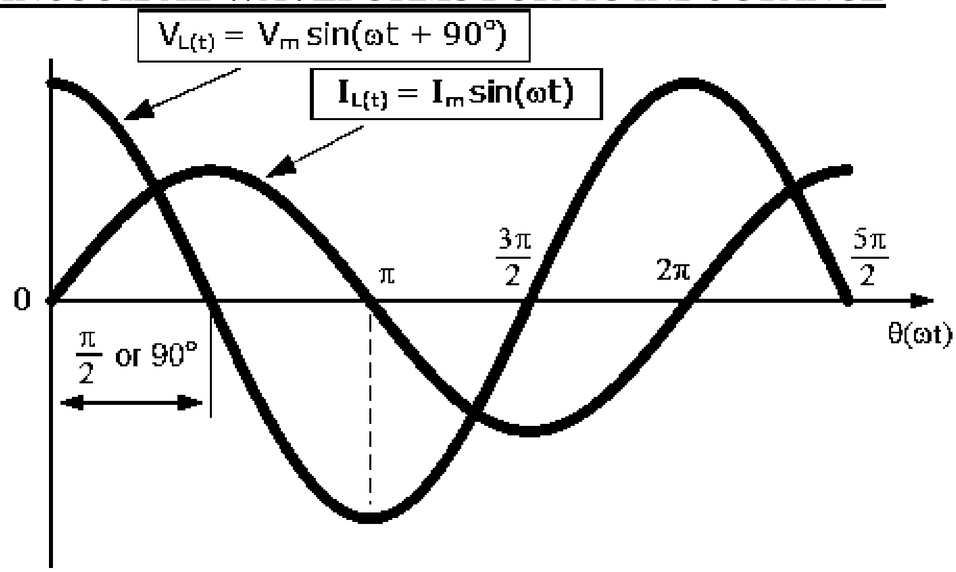
$V(t) = V_{max} \sin \omega t$.

When the switch is closed, this sinusoidal voltage will cause a current to flow, and rise from zero to its maximum value. This rise or change in the current will induce a magnetic field within the coil which in turn will oppose or restrict this change in the current.

But before the current has had time to reach its maximum value as it would in a DC circuit, the voltage changes polarity causing the current to change direction. This change in the other direction once again being delayed by the self-induced back emf in the coil, and in a circuit containing a pure inductance only, the current is delayed by $90^o$.

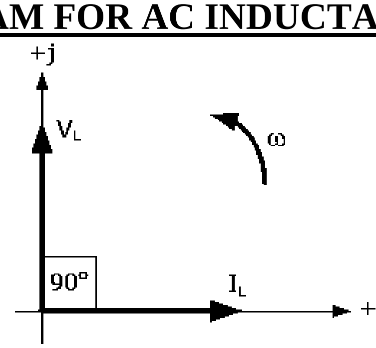
The applied voltage reaches its maximum positive value a quarter ( $1/4f$ ) of a cycle earlier than the current reaches its maximum positive value, in other words, a voltage applied to a purely inductive circuit "LEADS" the current by a quarter of a cycle or $90^o$ as shown below.

## SINUSOIDAL WAVEFORMS FOR AC INDUCTANCE




This effect can also be represented by a Phasor diagram were in a purely inductive circuit the voltage "LEADS" the current by $90^o$. But by using the voltage as our reference, we can also say that the current "LAGS" the voltage by one quarter of a cycle or $90^o$ as shown in the vector diagram below.

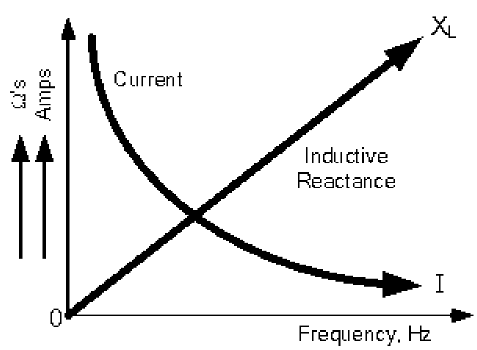
## PHASOR DIAGRAM FOR AC INDUCTANCE



So, for a pure loss less inductor, $V_L$ "leads" $I_L$ by $90^o$, or we can say that $I_L$ "lags" $V_L$ by $90^o$.

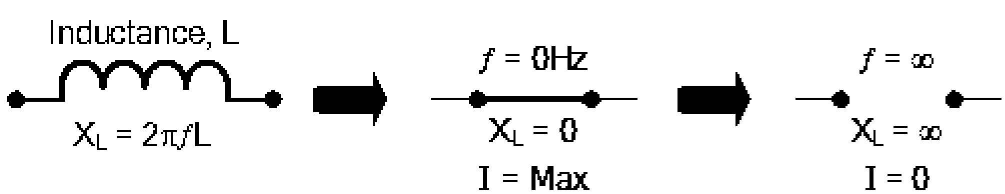
There are many different ways to remember the phase relationship between the voltage and current flowing through a pure inductor circuit, but one very simple and easy to remember way is to use the mnemonic expression "ELI" (pronounced ELLIE as in the girl's name). ELI stands for Electromotive force first in an AC inductance, L before the current I. In other words, voltage before the current in an inductor, E, L, I equal "ELI", and whichever phase angle the voltage starts at, this expression always holds true for a pure inductor circuit.

## INDUCTIVE REACTANCE AGAINST FREQUENCY:




The inductive reactance of an inductor increases as the frequency across it increases therefore inductive reactance is proportional to frequency ($X_L \alpha f$) as the back emf generated in the inductor is equal to its inductance multiplied by the rate of change of current in the inductor. Also, as the frequency increases the current flowing through the inductor also reduces in value.

We can present the effect of very low and very high frequencies on the reactance of a pure AC Inductance as follows:




In an AC circuit containing pure inductance the following formula applies:

$$\text{Current, I} = \frac{\text{Voltage}}{\text{Opposition to current flow}} = \frac{V}{X_L}$$

So how did we arrive at this equation? Well the self-induced emf in the inductor is determined by Faraday's Law that produces the effect of self-induction in the inductor due to the rate of change of the current and the maximum value of the induced emf will correspond to the maximum rate of change. Then the voltage in the inductor coil is given as:

$$V_{L(t)} = L\frac{di_{L(t)}}{dt}$$

$$\text{If,}\quad i_{L(t)} = I_{max}\sin(\omega t)\quad \text{then:}$$

$$V_{L(t)} = L\frac{d}{dt}I_{max}\sin(\omega t+\theta)$$

$$= \omega L I_{max}\cos(\omega t+\theta)$$

Then the voltage across an AC inductance will be defined as:

$$V_L = \omega L I_{max}\sin(\omega t + 90^{o})$$

Where: $V_L = I\omega L$ which is the voltage amplitude and $\theta = +90^{o}$ which is the phase difference or phase angle between the voltage and current.

## IN THE PHASOR DOMAIN:

In the Phasor domain the voltage across the coil is given as:

$$V_L = j\omega L I$$

$$\text{where:}\ j\omega L = jX_L = 2\pi f L = \text{IMPEDANCE, Z}$$

and in Polar Form this would be written as: $X_L\angle 90^{o}$ where:


## EXAMPLE - 1:

Find the current which will flow through a coil of negligible resistance and inductance 0.04 henry, when connected to 200V, 50 C/S supply. What will be the current if the frequency is (a) Halved (b) Doubled?

**Solution:**

R = 0 Ω, L= 0.04 H, F= 50 C/S, I=?

XL = 2 π F L = 2 x 3.142 x 50 x 0.04 = 12.57 Ω

$I = \frac{V}{X_L} = \frac{200}{12.57}$ = 15.91 A

(a) Frequency is halved i.e. 25 C/S

XL = 2 π F L = 2 x 3.142 x 25 x 0.04 = 6.285 Ω

$I = \frac{V}{X_L} = \frac{200}{6.285}$ = 31.82 A i.e. Doubled from the previous value

(b) Frequency is doubled i.e. 100 C/S

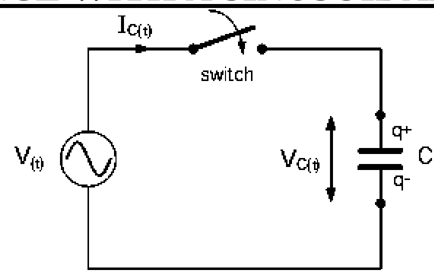
$X_L = 2\ \pi\ F\ L = 2 \times 3.142 \times 100 \times 0.04 = 25.14\ \Omega$

$I = \frac{V}{X_L} = \frac{200}{25.14} = 7.955$ A i.e. halved from the previous value

## PURE CAPACITIVE AC CIRCUIT:

When an alternating sinusoidal voltage is applied to the plates of an AC capacitor, the capacitor is charged firstly in one direction and then in the opposite direction, changing polarity at the same rate as the AC supply voltage. This instantaneous change in voltage across the capacitor is opposed by the fact that it takes a certain amount of time to deposit (or release) this charge onto the plates and is given by V = Q/C.

## AC CAPACITANCE WITH A SINUSOIDAL SUPPLY:




When the switch is closed in the circuit above, a high current will start to flow into the capacitor as there is no charge on the plates at $T = 0$. The sinusoidal supply voltage, V is increasing in a positive direction at its maximum rate as it crosses the zero reference axes at an instant in time given as $0^o$. Since the rate of change of the potential difference across the plates is now at its maximum value, the flow of current through the capacitor will also be at its maximum rate as the maximum number of electrons are moving from one plate to the other.

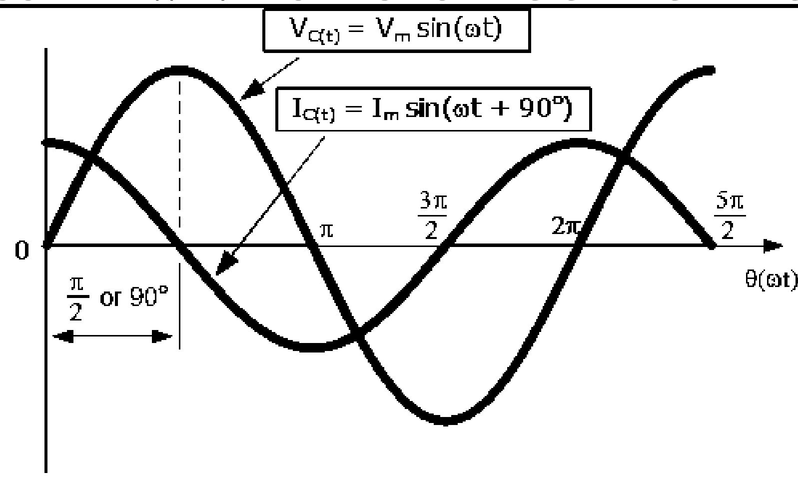
As the sinusoidal supply voltage reaches its $90^o$ point on the waveform it begins to slow down and for a very brief instant in time the potential difference across the plates is neither increasing nor decreasing therefore the current decreases to zero as there is no rate of voltage change. At this $90^o$point the potential difference across the capacitor is at its maximum ($V_{max}$), no current flows into the capacitor as the capacitor is now fully charged.

At the end of this instant in time the supply voltage begins to decrease in a negative direction down towards the zero-reference line at $180^o$. Although the supply voltage is still positive in nature the capacitor starts to discharge some of its excess electrons on its plates in an effort to maintain a constant voltage, resulting current flowing in the capacitor, in the opposite or negative direction.

When the supply voltage waveform crosses the zero-reference axis point at instant $180^o$, the rate of change or slope of the sinusoidal supply voltage is at its maximum but in a negative direction, consequently the current flowing through the capacitor is also at its maximum rate at that instant. Also, at this $180^o$ point the potential difference across the plates is zero as the amount of charge is equally distributed between the two plates.

Then during this first half cycle $0^o$ to $180^o$, the applied voltage reaches its maximum positive value a quarter $(1/4f)$ of a cycle after the current reaches its maximum positive value, in other words, a voltage applied to a purely capacitive circuit "LAGS" the current by a quarter of a cycle or $90^o$ as shown below.

## SINUSOIDAL WAVEFORMS FOR AC CAPACITANCE:



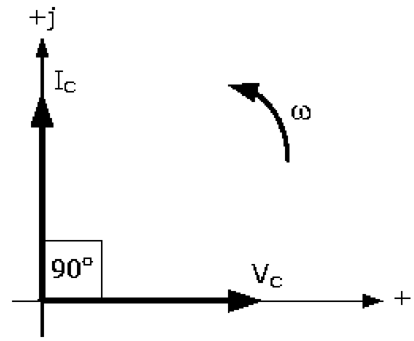

During the second half cycle $180^o$ to $360^o$, the supply voltage reverses direction and heads towards its negative peak value at $270^o$. At this point the potential difference across the plates is neither decreasing nor increasing and the current decreases to zero. The potential difference across the capacitor is at its maximum negative value, no current flows into the capacitor and it becomes fully charged the same as at its $90^o$ point but in the opposite direction.

As the negative supply voltage begins to increase in a positive direction towards the $360^o$ point on the zero reference line, the fully charged capacitor must now loose some of its excess electrons to maintain a constant voltage as before and starts to discharge itself until the supply voltage reaches zero at $360^o$ at which the process of charging and discharging starts over again.

From the voltage and current waveforms and description above, we can see that the current is always leading the voltage by 1/4 of a cycle or $\pi/2 = 90^o$ "out-of-phase" with the potential difference across the capacitor because of this charging and discharging process. Then the phase relationship between the voltage and current in an AC capacitance circuit is the exact opposite to that of an AC Inductance we saw in the previous tutorial.

This effect can also be represented by a Phasor diagram where in a purely capacitive circuit the voltage "LAGS" the current by $90^o$. But by using the voltage as our reference, we can also say that the current "LEADS" the voltage by one quarter of a cycle or $90^o$ as shown in the vector diagram below.

## PHASER DIAGRAM FOR AC CAPACITANCE:




So, for a pure capacitor, $V_C$ "lags" $I_C$ by $90^o$, or we can say that $I_C$ "leads" $V_C$ by $90^o$.

There are many different ways to remember the phase relationship between the voltage and current flowing through a pure AC capacitance circuit, but one very simple and easy to remember way is to use the mnemonic expression called "ICE". ICE stands for current I first in an AC capacitance, C before Electromotive force. In other words, current before the voltage in a capacitor, I, C, E equals "ICE", and whichever phase angle the voltage starts at, this expression always holds true for a pure AC capacitance circuit.

## CAPACITIVE REACTANCE:

Capacitors oppose changes in voltage with the flow of electrons through the capacitor being directly proportional to the rate of voltage change across its plates as the capacitor charges and discharges. Unlike a resistor where the opposition to current flow is its actual resistance, the opposition to current flow in a capacitor is called Reactance**.**

Like resistance, reactance is measured in Ohm's but is given the symbol X to distinguish it from a purely resistive R value and as the component in question is a capacitor, the reactance of a capacitor is called Capacitive Reactance**,** ($X_C$) which is measured in Ohms.

Since capacitors pass current through them in proportion to the rate of voltage change, the faster the voltage changes the more current they will pass. Likewise, the slower the voltage changes the less current they will pass. This means then that the reactance of an AC capacitor is "inversely proportional" to the frequency of the supply as shown.

$$X_C = \frac{1}{2\pi f C}$$

Where: $X_C$ is the Capacitive Reactance in Ohms, $f$ is the frequency in Hertz and C is the AC capacitance in Farads, symbol F.
When dealing with AC capacitance, we can also define capacitive reactance in terms of radians, where Omega, ω equals $2\pi f$.

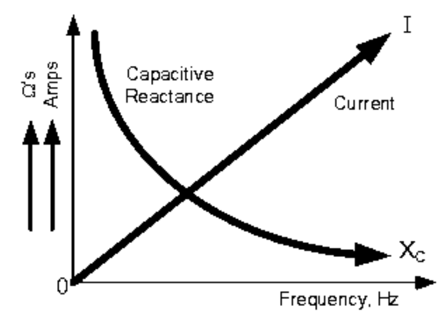
$$X_C = \frac{1}{\omega C}$$

From the above formula we can see that the value of capacitive reactance and therefore its overall impedance (in Ohms) decreases towards zero as the frequency increases acting like a short circuit. Likewise, as the frequency approaches zero or DC, the capacitors reactance increases to infinity, acting like an open circuit which is why capacitors block DC.

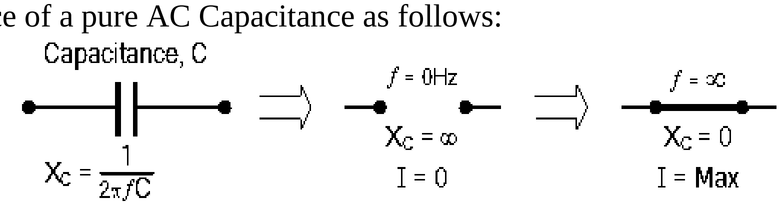
The relationship between capacitive reactance and frequency is the exact opposite to that of inductive reactance, ($X_L$) we saw in the previous tutorial. This means that capacitive reactance is "inversely proportional to frequency" and has a high value at low frequencies and a low value at higher frequencies.

## CAPACITIVE REACTANCE AGAINST FREQUENCY:

Capacitive reactance of a capacitor decreases as the frequency across its plates increases. Therefore, capacitive reactance is inversely proportional to frequency. Capacitive reactance opposes current flow but the electrostatic charge on the plates (its AC capacitance value) remains constant. This means it becomes easier for the capacitor to fully absorb the change in charge on its plates during each half cycle. Also, as the frequency increases the current flowing through the capacitor increases in value because the rate of voltage change across its plates increases.




We can present the effect of very low and very high frequencies on the reactance of a pure AC Capacitance as follows:



In an AC circuit containing pure capacitance the current flowing through the capacitor is given as:

$$I_{C(t)} = \frac{dq}{dt} \quad \text{where: } q = CV_C = CV_{max}\sin(\omega t)$$

$$\therefore I_{C(t)} = \frac{d}{dt} CV_{max}\sin(\omega t) = \omega CV_{max}\cos(\omega t)$$

$$\text{If: } I_{max} = \frac{V_{max}}{Xc} \quad \text{where: } Xc = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

$$\text{then: } I_{max} = \omega CV_{max}$$

Therefore, the rms current flowing through an AC capacitance will be defined as:

$$I_{C(t)} = I_{max}\sin\left(\omega t + 90^{o}\right)$$

Where: $I_C = V/(\omega C)$ which is the current amplitude and $\theta = +90^{o}$ which is the phase difference or phase angle between the voltage and current. For a purely capacitive circuit, $I_c$ leads $V_c$ by $90^{o}$, or $V_c$ lags $I_c$ by $90^{o}$.

## PHASOR DOMAIN:

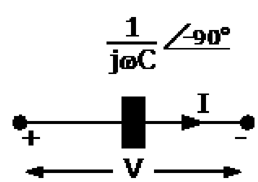
In the Phasor domain the voltage across the plates of an AC capacitance will be:

$$V_C = \frac{1}{j\omega C} \times I_C$$

$$\text{where: } \frac{1}{j\omega C} = jX_C = \frac{1}{2\pi f C} = \text{IMPEDANCE, Z}$$

In Polar Form this would be written as: $X_C\angle{-90^{o}}$ where:

$$X_C\angle\theta = \frac{V_C\angle 0^{o}}{I_C\angle{+90^{o}}}$$



**EXAMPLE**

Find the current taken by a 20µF capacitor when connected to a 240V, 50 Hertz supply. What alteration in current will occur if the frequency is (a) Doubled (b) Halved, the supply voltage being kept constant.

Solution:

C= 20µF, V=240V, F=50 Hz, I=?

$$XC = \frac{10^6}{2\pi F c} = \frac{10^6}{2 \times 3.142 \times 50 \times 20} = 159.134\ \Omega$$

$$I = \frac{V}{Xc} = \frac{240}{159.134} = 1.508\ A$$

(a) When frequency is doubled i.e. 100 Hz

$$XC = \frac{10^6}{2\pi F c} = \frac{10^6}{2 \times 3.142 \times 100 \times 20} = 79.567\ \Omega$$

$$I = \frac{V}{Xc} = \frac{240}{79.567} = 3.016\ A$$

(b) When frequency is halved i.e. 25 Hz

$$XC = \frac{10^6}{2\pi F c} = \frac{10^6}{2 \times 3.142 \times 25 \times 20} = 318.268\Omega$$

$$I = \frac{V}{Xc} = \frac{240}{318.268} = 0.754\ A$$

## 8.2 AC THROUGH A SERIES R + L CIRCUIT:

We have seen above that the current flowing through a purely inductive coil lags the voltage by $90^o$ and when we say a purely inductive coil we mean one that has no ohmic resistance and therefore, no $I^2R$ losses. But in the real world, it is impossible to have a purely AC Inductance only.

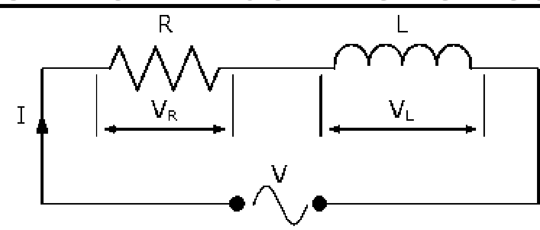
All electrical coils, relays, solenoids and transformers will have a certain amount of resistance no matter how small associated with the coil turns of wire being used. This is because copper wire has resistivity. Then we can consider our inductive coil as being one that has a resistance, R in series with an inductance, L producing what can be loosely called an “impure inductance”.

If the coil has some “internal” resistance then we need to represent the total impedance of the coil as a resistance in series with an inductance and in an AC circuit that contains both inductance, L and resistance, $R_{th}$ voltage, V across the combination will be the Phasor sum of the two component voltages, $V_R$ and $V_L$.

This means then that the current flowing through the coil will still lag the voltage, but by an amount less than $90^o$ depending upon the values of $V_R$ and $V_L$, the Phasor sum. The new angle between the voltage and the current waveforms gives us their Phase Difference which as we know is the phase angle of the circuit given the Greek symbol phi, Φ.

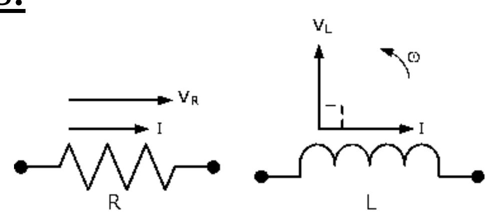
Consider the circuit below was a pure non-inductive resistance; R is connected in series with a pure inductance, L.

## SERIES RESISTANCE-INDUCTANCE CIRCUIT:



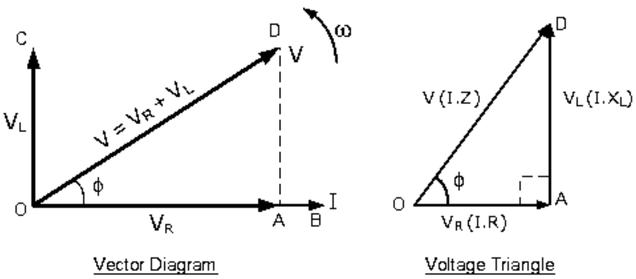

In the RL series circuit above, we can see that the current is common to both the resistance and the inductance while the voltage is made up of the two component voltages, $V_R$ and $V_L$. The resulting voltage of these two components can be found either mathematically or by drawing a vector diagram. To be able to produce the vector diagram a reference or common component must be found and in a series AC circuit the current is the reference source as the same current flows through the resistance and the inductance. The individual vector diagrams for a pure resistance and a pure inductance are given as:

## VECTOR DIAGRAMS FOR THE TWO PURE COMPONENTS:




We can see from above and from our previous tutorial about AC Resistance that the voltage and current in a resistive circuit are both in phase and therefore vector $V_R$ is drawn superimposed to scale onto the current vector. Also, from above it is known that the current lags the voltage in an AC inductance (pure) circuit therefore vector $V_L$ is drawn $90^o$ in front of the current and to the same scale as $V_R$ as shown above.

## VECTOR DIAGRAM OF THE RESULTANT VOLTAGE:




From the vector diagram above, we can see that line OB is the horizontal current reference and line OA is the voltage across the resistive component which is in-phase with the current. Line OC shows the inductive voltage which is $90^o$ in front of the current therefore it can still be seen that the current lags the purely inductive voltage by $90^o$. Line OD gives us the resulting supply voltage. Then:

- V, is equals the rms value of the applied voltage.
- I, is equals the rms value of the series current.
- $V_{R,}$ is equals the I.R voltage drop across the resistance which is in-phase with the current.
- $V_L$ is equals the I.$X_L$ voltage drop across the inductance which leads the current by $90^o$.

As the current lags the voltage in a pure inductance by exactly $90^o$ the resultant Phasor diagram drawn from the individual voltage drops $V_R$ and $V_L$ represents a right-angled voltage triangle shown above as OAD. Then we can also use Pythagoras's theorem to mathematically find the value of this resultant voltage across the resistor/inductor (RL) circuit.

As $V_R$ = I.R and $V_L$ = I.$X_L$ the applied voltage will be the vector sum of the two as follows:

$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(I.R)^2 + (I.X_L)^2}$$

$$\therefore I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

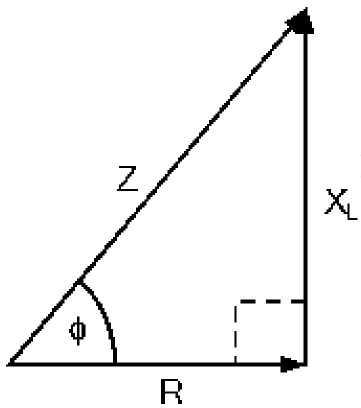
The quantity $\sqrt{R^2 + X_L^2}$ represents the impedance**,** Z of the circuit.

## THE IMPEDANCE OF AN AC INDUCTANCE:

Impedance, **Z** is the "TOTAL" opposition to current flowing in an AC circuit that contains both Resistance, (the real part) and Reactance (the imaginary part). Impedance also has the units of Ohms, Ω. Impedance depends upon the frequency, ω of the circuit as this affects the circuits reactive components and in a series circuit all the resistive and reactive impedances add together.

Impedance can also be represented by a complex number, $Z = R + jX_L$ but it is not a Phasor, it is the result of two or more Phasor combined together. If we divide the sides of the voltage triangle above by I.

## THE RL IMPEDANCE TRIANGLE:




$$\text{Impedance, } Z = \frac{V}{I}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\therefore Z^2 = \left(R^2 + X_L^2\right)$$

Then: $(\text{Impedance})^2 = (\text{Resistance})^2 + (\ J \ \text{Reactance}\ )^2$ where J represents the $90^o$ phase shift. This means that the positive phase angle, θ between the voltage and current is given as.

## PHASE ANGLE:

$$Z^2 = R^2 + X_L^2$$

$$\cos\phi = \frac{R}{Z}$$

$$\sin\phi = \frac{X_L}{Z}$$

$$\tan\phi = \frac{X_L}{R}$$

While our example above represents a simple non-pure AC inductance, if two or more inductive coils are connected together in series or a single coil is connected in series with many non-inductive resistances, then the total resistance for the resistive elements would be equal to: $R_1 + R_2 + R_3$ etc., giving a total resistive value for the circuit.

Likewise, the total reactance for the inductive elements would be equal to: $X_1 + X_2 + X_3$ etc., giving a total reactance value for the circuit. This way a circuit containing many chokes, coils and resistors can be easily reduced down to an impedance value, Z comprising of a single resistance in series with a single reactance, $Z^2 = R^2 + X^2$.

**POWER:**

Let $I = \frac{V}{Z}$

$V = I\,R$

$P = I^2\,R$

$P = I.\,I.\,R$

$P = I\,.\frac{V}{Z}\,.\,R$

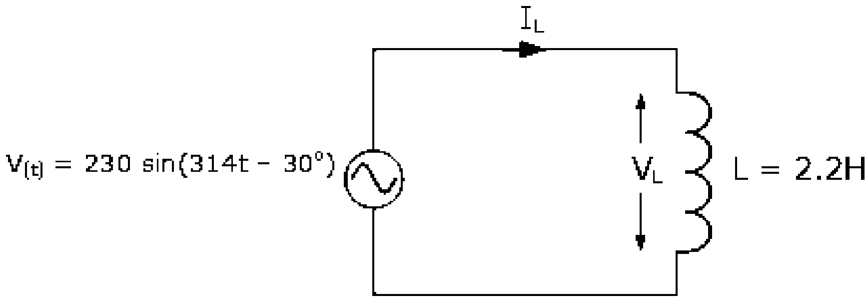
$P = V\,I\,\frac{R}{Z}$

$P = V\,I\,\cos\theta$

## AC INDUCTANCE

## EXAMPLE NO.1:

In the following circuit, the supply voltage is defined as:
$V_{(t)} = 230 \sin(314t - 30^o)$ and L = 2.2H. Determine the value of the current flowing through the coil and draw the resulting Phasor diagram.




The voltage across the coil will be the same as the supply voltage. Converting this time domain value into polar form gives us:

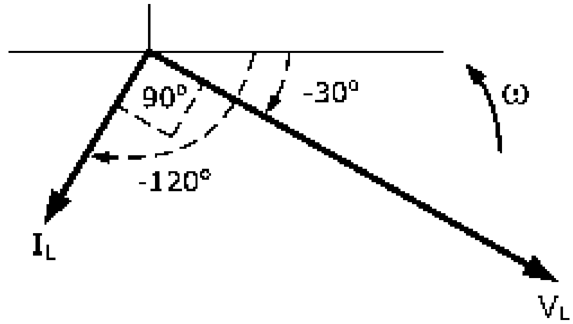
$V_L = 230\angle{-30^o}$ (v).

The inductive reactance of the coil is:

$X_L = \omega L = 314 \times 2.2 = 690\Omega.$

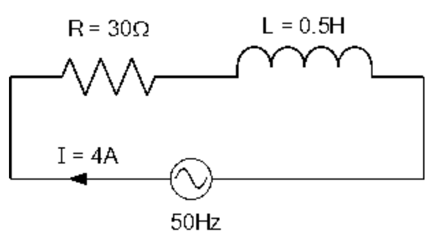
Then the current flowing through the coil can be found using Ohms law as:

$$I_L = \frac{V_L}{jX_L} = \frac{230\angle{-30^0}}{690\angle 90^0} = 0.33\angle{-120^0}\ \text{(A)}$$




## AC INDUCTANCE
## EXAMPLE NO.2:

A coil has a resistance of 30Ω and an inductance of 0.5H. If the current flowing through the coil is 4amps; what will be the value of the impedance, supply voltage and power if its frequency is 50Hz.




$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157\Omega$$

$$Z = \sqrt{R^2 + X_L^2}$$
$$Z = \sqrt{30^2 + 157^2}$$
$$Z = 159.8\Omega$$

Then the voltage drops across each component is calculated as:

$$V_S = I.Z = 4 \times 159.8 = 640v$$

$$V_R = I.R = 4 \times 30 = 120v$$

$$V_L = I.X_L = 4 \times 157 = 628v$$

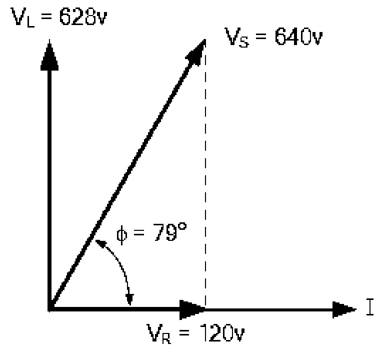
The phase angle between the current and supply voltage is calculated as:

$$\text{Cos } \theta = \frac{R}{Z} = \frac{30}{159.8} = 0.188$$

$$\theta = \cos^{-1} 0.188 = 79^o$$

$$P = V\,I\cos\theta = 640 \times 4 \times 0.188 = 481.28\text{ W}$$

The Phasor diagram will be.




## AC THROUGH A SERIES R + C CIRCUIT:

We have seen from above that the current flowing through a pure AC capacitance leads the voltage by $90^o$. But in the real world, it is impossible to have a pure AC Capacitance as all capacitors will have a certain amount of internal resistance across their plates giving rise to a leakage current.

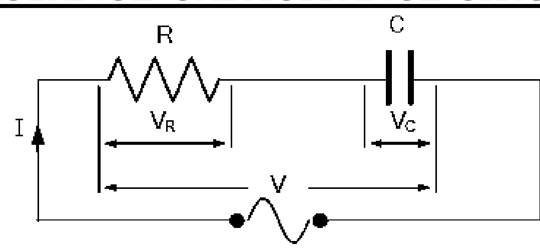
Then we can consider our capacitor as being one that has a resistance, R in series with a capacitance, C producing what can be loosely called an "impure capacitor".

If the capacitor has some "internal" resistance then we need to represent the total impedance of the capacitor as a resistance in series with a capacitance and in an AC circuit that contains both capacitance and resistance, R the voltage Phasor, V across the combination will be equal to the Phasor sum of the two component voltages, $V_R$ and $V_C$.

This means then that the current flowing through the capacitor will still lead the voltage, but by an amount less than $90^o$ depending upon the values of R and C giving us a Phasor sum with the corresponding phase angle between them given by the Greek symbol phi, Φ.

Consider the series RC circuit below where an Ohmic resistance, R is connected in series with a pure capacitance, C.

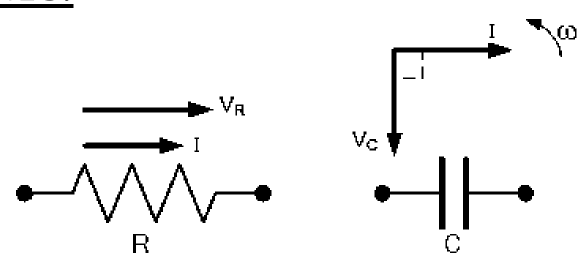
## SERIES RESISTANCE-CAPACITANCE CIRCUIT:




In the RC series circuit above, we can see that the current flowing through the circuit is common to both the resistance and capacitance, while the voltage is made up of the two component voltages, $V_R$ and $V_C$. The resulting voltage of these two components can be found mathematically but since vectors $V_R$ and $V_C$ are $90^o$ out-of-phase, they can be added vectors by constructing a vector diagram.

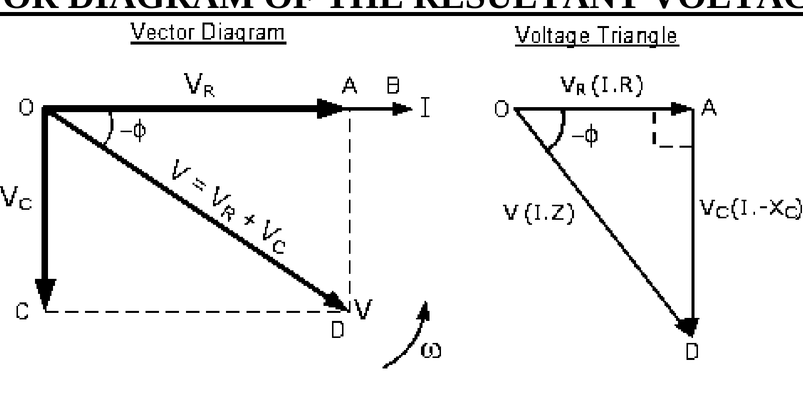
To be able to produce a vector diagram for an AC capacitance a reference or common component must be found. In a series AC circuit, the current is common and can therefore be used as the reference source because the same current flows through the resistance and capacitance. The individual vector diagrams for a pure resistance and a pure capacitance are given as:

## VECTOR DIAGRAMS FOR THE TWO PURE COMPONENTS:




Both the voltage and current vectors for an AC Resistance are in phase with each other and therefore the voltage vector $V_R$ is drawn superimposed to scale onto the current vector. Also, we know that the current leads the voltage (ICE) in a pure AC capacitance circuit, therefore the voltage vector $V_C$ is drawn $90^o$ behind (lagging) the current vector and to the same scale as $V_R$ as shown.

## VECTOR DIAGRAM OF THE RESULTANT VOLTAGE:




In the vector diagram above, line OB represents the horizontal current reference and line OA is the voltage across the resistive component which is in-phase with the current. Line OC shows the capacitive voltage which is

$90^o$ behind the current therefore it can still be seen that the current leads the purely capacitive voltage by $90^o$. Line OD gives us the resulting supply voltage.

As the current leads the voltage in a pure capacitance by $90^o$ the resultant Phasor diagram drawn from the individual voltage drops $V_R$ and $V_C$ represents a right-angled voltage triangle shown above as OAD. Then we can also use Pythagoras's theorem to mathematically find the value of this resultant voltage across the resistor/capacitor (RC) circuit.

As $V_R = I.R$ and $V_C = I.X_C$ the applied voltage will be the vector sum of the two as follows.

$$V^2 = \mathbf{V_R^2} + \mathbf{V_C^2}$$

$$V_R = I\,.\,R \text{ and } V_C = I\,.\,X_C$$

$$V^2 = (I.\,R)^2 + (I\,.X_C)^2$$

$$V = \sqrt{(\mathbf{I}.\mathbf{R})^2 + (\mathbf{I}.\mathbf{X_C})^2}$$

$$I = \frac{\mathbf{V}}{\mathbf{Z}}$$

The quantity   represents the impedance**,** Z of the circuit.

## THE IMPEDANCE OF AN AC CAPACITANCE:

Impedance, **Z** which has the units of Ohms, Ω's is the "TOTAL" opposition to current flowing in an AC circuit that contains both Resistance, (the real part) and Reactance (the imaginary part). Purely resistive impedance will have a phase angle of $0^o$ while purely capacitive impedance will have a phase angle of $-90^o$.

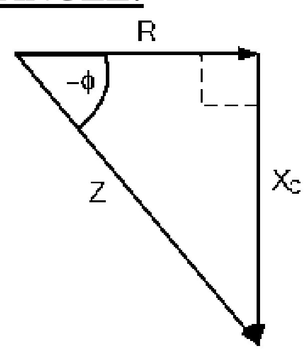
However, when resistors and capacitors are connected together in the same circuit, the total impedance will have a phase angle somewhere between $0^o$ and $90^{o,}$ depending upon the value of the components used. Then the impedance of our simple RC circuit shown above can be found by using the impedance triangle.

## THE RC IMPEDANCE TRIANGLE:

$$\text{Impedence} = Z = \frac{\mathbf{V}}{\mathbf{I}}$$

$$Z =$$

$$Z^2 = R^2 + \mathbf{X_C^2}$$




Then: $(\text{Impedance})^2 = (\text{ Resistance })^2 + (\text{ Reactance })^2$

This means then by using Pythagoras's theorem the negative phase angle, θ between the voltage and current is calculated as.

**PHASE ANGLE:**

$$Z^2 = R^2 + X_C^2$$

$$\cos\Phi = \frac{R}{Z}$$

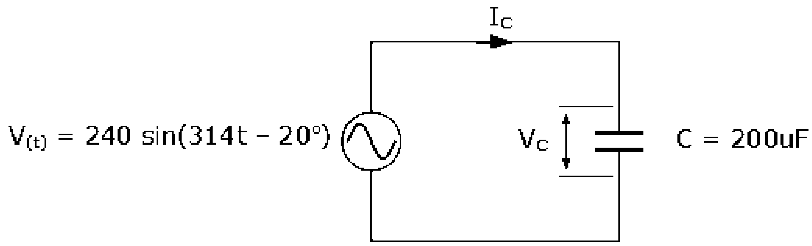
$$\sin\Phi = \frac{X_C}{Z}$$

$$\tan\Phi = \frac{X_C}{R}$$

## AC CAPACITANCE EXAMPLE NO.1:

A single-phase sinusoidal AC supply voltage defined as:

$V_{(t)} = 240\sin(314t - 20^o)$ is connected to a pure AC capacitance of 200uF. Determine the value of the current flowing through the capacitor and draw the resulting Phasor diagram.




The voltage across the capacitor will be the same as the supply voltage. Converting this time domain value into polar form gives us:

$V_C = 240\angle{-20^o}$ (v).

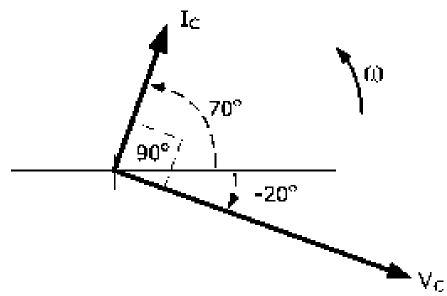
The capacitive reactance will be: $X_C = 1/(\omega.200\mu F)$.

Then the current flowing through the capacitor can be found using Ohms law as:

$$X_C = \frac{1}{j\omega C} = \frac{1}{314 \times 200\mu F} = 16\angle{-90^\circ}$$

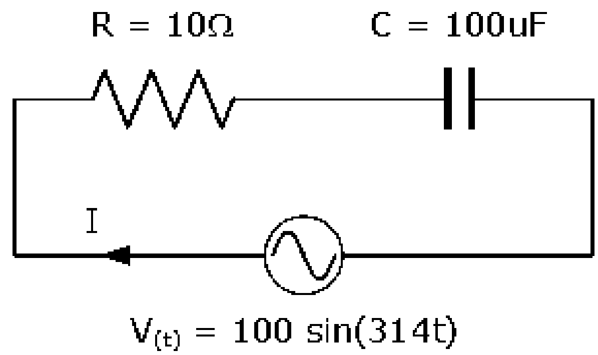
$$I_C = \frac{V_C}{jX_C} = \frac{240\angle{-20^\circ}}{16\angle{-90^\circ}} = 15\angle 70^\circ \ (A)$$

With the current leading the voltage by $90^o$ in an AC capacitance circuit the Phasor diagram will be.




## AC CAPACITANCE EXAMPLE NO.2:

A capacitor which has an internal resistance of 10Ω's and a capacitance value of 100uF is connected to a supply voltage given as $V_{(t)}$ = 100 sin (314t). Calculate the current flowing through the capacitor, impedance and power taken. Also construct a voltage triangle showing the individual voltage drops.




The capacitive reactance and circuit impedance is calculated as:

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100uF} = 31.85\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 31.85^2} = 33.4\Omega$$

Then the current flowing through the capacitor and the circuit is given as:

$$I = \frac{V_C}{Z} = \frac{100}{33.4} = 3Amps$$

The phase angle between the current and voltage is calculated from the impedance triangle above as:

$$\cos\theta = \frac{R}{Z} = \frac{10}{33.4} = 0.299$$

$$\Theta = \cos^{-1} 0.299 = 72.6^{\circ}$$

$$P = V\,I\cos\theta$$

$$P = 100 \times 3 \times 0.299 = 89.82\ W$$

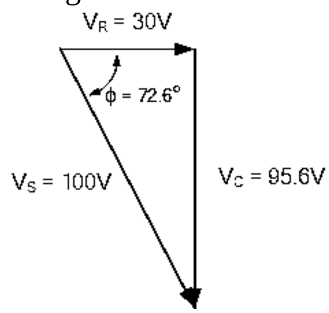
Then the individual voltage drops around the circuit are calculated as:

$$V_R = I \times R = 3 \times 10 = 30V$$

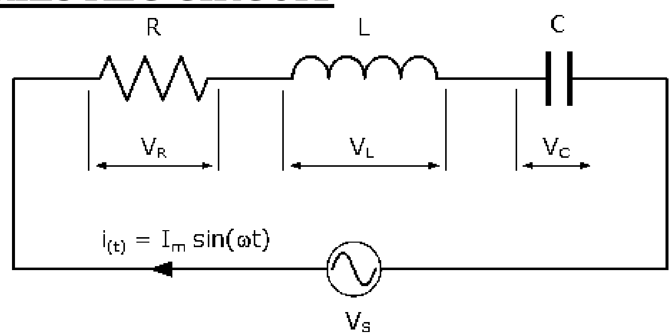
$$V_C = I \times X_C = 3 \times 31.85 = 95.6V$$

$$V_S = \sqrt{V_R^2 + V_C^2} = \sqrt{30^2 + 95.6^2} = 100V$$

Then the resultant voltage triangle will be.




## 8.4 SERIES RLC CIRCUIT

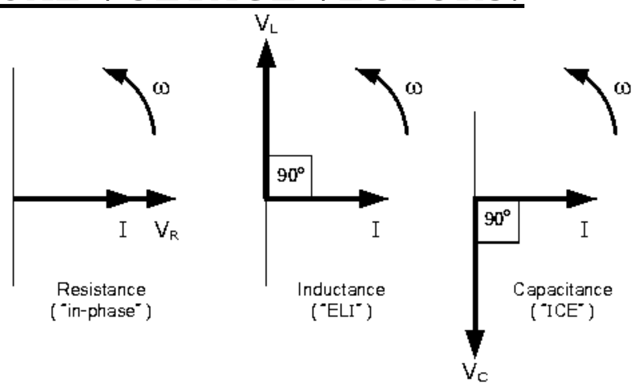



The series RLC circuit above has a single loop with the instantaneous current flowing through the loop being the same for each circuit element. Since the inductive and capacitive reactance's are a function of frequency, the sinusoidal response of a series RLC circuit will vary with the applied frequency, ($f$). Therefore, the individual voltage drops across each circuit element of R, L and C element will be "out-of-phase" with each other as defined by:

- $i_{(t)} = I_{max} \sin(\omega t)$
- The instantaneous voltage across a pure resistor, $V_R$ is "in-phase" with the current.
- The instantaneous voltage across a pure inductor, $V_L$ "leads" the current by $90^o$
- The instantaneous voltage across a pure capacitor, $V_C$ "lags" the current by $90^o$
- Therefore, $V_L$ and $V_C$ are $180^o$ "out-of-phase" and in opposition to each other.

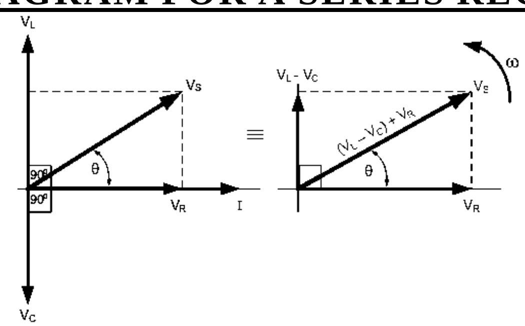
Then the amplitude of the source voltage across all three components in a series RLC circuit is made up of the three individual component voltages, $V_R$, $V_L$ and $V_C$ with the current common to all three components. The vector diagrams will therefore have the current vector as their reference with the three voltage vectors being plotted with respect to this reference as shown below.

## INDIVIDUAL VOLTAGE VECTORS:




This means then that we cannot simply add together $V_R$, $V_L$ and $V_C$ to find the supply voltage, $V_S$ across all three components as all three voltage vectors point in different directions with regards to the current vector. Therefore, we will have to find the supply voltage, $V_S$ as the Phasor Sum of the three component voltages combined together vector sum.

## PHASOR DIAGRAM FOR A SERIES RLC CIRCUIT:




We can see from the Phasor diagram on the right hand side above that the voltage vectors produce a rectangular triangle, comprising of hypotenuse $V_S$, horizontal axis $V_R$ and vertical axis $V_L - V_C$, voltage triangle and we can therefore use Pythagoras's theorem on this voltage triangle to mathematically obtain the value of $V_S$ as shown.

## VOLTAGE TRIANGLE FOR A SERIES RLC CIRCUIT:

$$V_S^2 = V_R^2 + (V_L - V_C)^2$$

$$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Please note that when using the above equation, the final reactive voltage must always be positive in value, that is the smallest voltage must always be taken away from the largest voltage we cannot have a negative voltage added to $V_R$ so it is correct to have $V_L – V_C$ or $V_C – V_L$. The smallest value from the largest otherwise the calculation of $V_S$ will be incorrect.

We know from above that the current has the same amplitude and phase in all the components of a series RLC circuit. Then the voltage across each component can also be described mathematically according to the current flowing through and the voltage across each element as.

$$V_R = iR\sin(\omega t + 0^0) = i.R$$

$$V_L = iX_L\sin(\omega t + 90^0) = i.j\omega L$$

$$V_C = iX_C\sin(\omega t - 90^0) = i.\frac{1}{j\omega C}$$

By substituting these values into Pythagoras's equation above for the voltage triangle will give us:

$$V_R = I.R \quad V_L = I.X_L \quad V_C = I.X_C$$

$$V_S = \sqrt{(I.R)^2 + (I.X_L - I.X_C)^2}$$

$$V_S = I.\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore V_S = I \times Z \quad \text{where: } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

So, we can see that the amplitude of the source voltage is proportional to the amplitude of the current flowing through the circuit. This proportionality constant is called the Impedance of the circuit which ultimately depends upon the resistance and the inductive and capacitive reactance's.

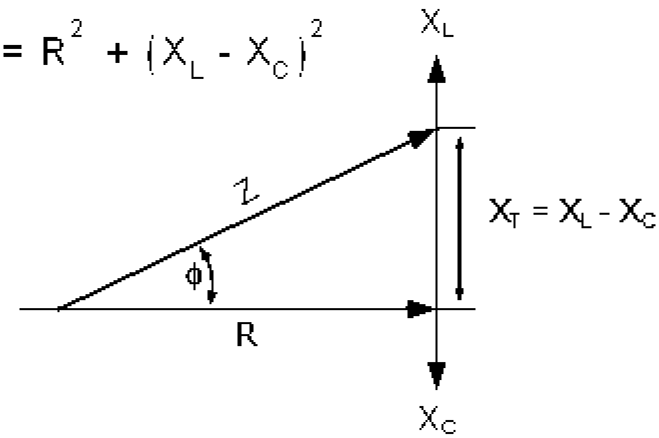
Then in the series RLC circuit above, it can be seen that the opposition to current flow is made up of three components, $X_L$, $X_C$ and R with the reactance, $X_T$ of any series RLC circuit being defined as: $X_T = X_L – X_C$ or $X_T = X_C – X_L$ with the total impedance of the circuit being thought of as the voltage source required to drive a current through it.

## THE IMPEDANCE OF A SERIES RLC CIRCUIT:

As the three vector voltages are out-of-phase with each other, $X_L$, $X_C$ and R must also be "out-of-phase" with each other with the relationship between R, $X_L$ and $X_C$ being the vector sum of these three components thereby giving us the circuits overall impedance, Z. This circuit impedance's can be drawn and represented by an **Impedance Triangle** as shown below.

## THE IMPEDANCE TRIANGLE FOR A SERIES RLC CIRCUIT:

$$Z^2 = R^2 + (X_L - X_C)^2$$




The impedance Z of a series RLC circuit depends upon the angular frequency, ω as do $X_L$ and $X_C$ If the capacitive reactance is greater than the inductive reactance, $X_C > X_L$ then the overall circuit reactance is capacitive giving a leading phase angle. Likewise, if the inductive reactance is greater than the capacitive reactance, $X_L > X_C$ then the overall circuit reactance is inductive giving the series circuit a lagging phase angle. If the two reactance's are the same and $X_L = X_C$ then the angular frequency at which this occurs is called the resonant frequency and produces the effect of resonance which we will look at in more detail in another tutorial.

Then the magnitude of the current depends upon the frequency applied to the series RLC circuit. When impedance, Z is at its maximum, the current is a minimum and likewise, when Z is at its minimum, the current is at maximum. So, the above equation for impedance can be re-written as:

$$\text{Impedance, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

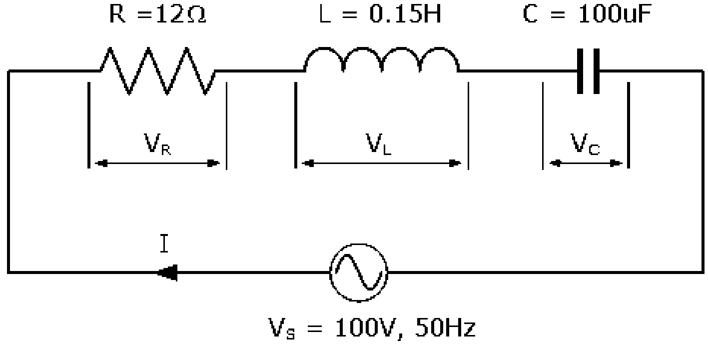
The phase angle, θ between the source voltage, $V_S$ and the current, i is the same as for the angle between Z and R in the impedance triangle. This phase angle may be positive or negative in value depending on whether the source voltage leads or lags the circuit current and can be calculated mathematically from the Ohmic values of the impedance triangle as:

$$\cos\phi = \frac{R}{Z} \quad \sin\phi = \frac{X_L - X_C}{Z} \quad \tan\phi = \frac{X_L - X_C}{R}$$

## SERIES RLC CIRCUIT

## EXAMPLE NO.1:

A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uFare connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuit's current, power factor and draw the voltage Phasor diagram.




Inductive Reactance, $X_L$.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance, $X_C$.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Circuit Impedance, Z

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuit Current, I

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.15\text{Amps}$$

Voltage across the Series RLC Circuit, $V_R$, $V_L$, $V_C$

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

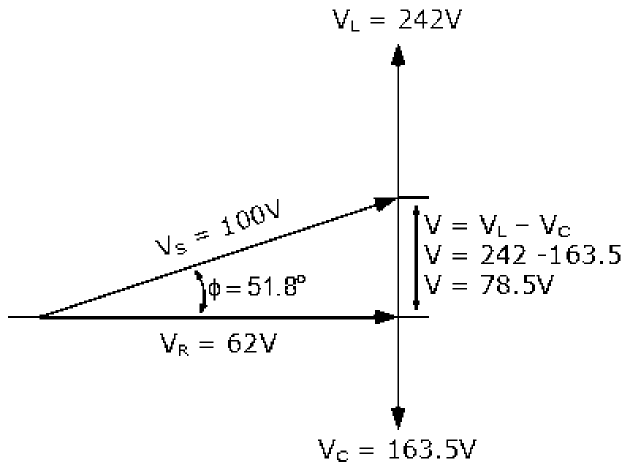
$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$

Circuits Power factor and Phase Angle, θ

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^{o} \text{ lagging}$$

Phasor Diagram




Since the phase angle θ is calculated as a positive value of 51.8$^o$ the overall reactance of the circuit must be inductive. As we have taken the current vector as our reference vector in a series RLC circuit, then the current “lags” the source voltage by 51.8$^o$.

## 8.6 ACTIVE & REACTIVE COMPONENT OF CURRENT:

Active component is that which is in phase with the applied voltage V i.e. I Cos θ, it is also known as watt full component, whereas reactive component is that which is in quadrature with V i.e. I Sin θ, it is also known as watt less component.

## ACTIVE, ACTUAL OR WORKING POWER:

Performs the actual work in creating heat, light, motion, or whatever else is required. It is measured in kilowatts (kW), it is denoted by P, mathematically;

$$P = V\, I \cos\theta$$

## REACTIVE POWER:

Doesn't do useful "work" but rather sustains the electromagnetic field. It is measured in kilovolt-amperes-reactive (KVAr), it is denoted with Q and mathematically:

$$Q = V\, I \sin\theta$$

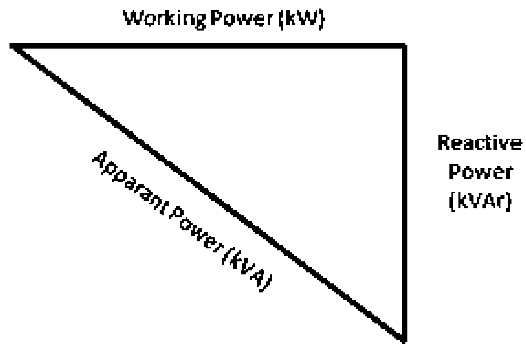
## APPARENT POWER:

These two types of power combine to create the Apparent Power. It is measured in kilovolt-amperes (kVA), it is denoted with S and mathematically:

$S = V\,I$

## RELATIONSHIP OF DIFFERENT POWERS:

These three types of power are related though the "Power Triangle" illustrated below:




Simple trigonometry gives us the relationship between all three Powers.

$$kVA^2 = kW^2 + kVAr^2$$

## POWER FACTOR:

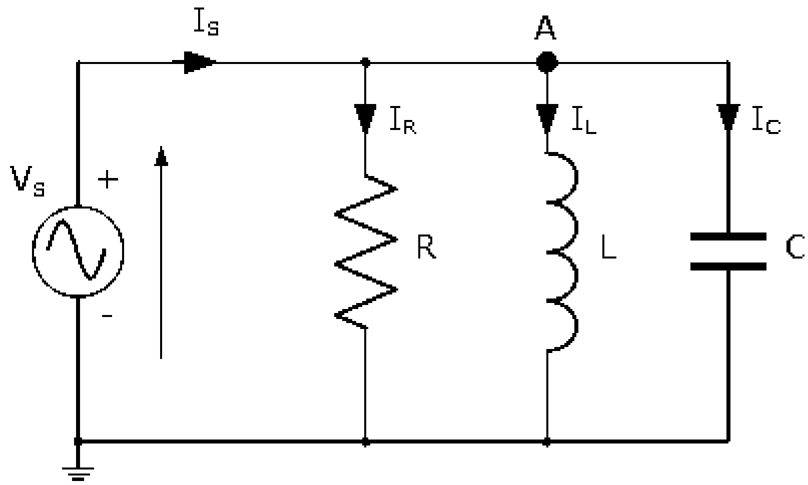
Power Factor is a measure of how effectively electrical power is being used in the conversion of current to work. The higher the Power Factor, the more effectively electrical power is utilized, conversely the lower the Power Factor the more ineffectively electrical power is utilized. Power Factor is defined as the ratio of working power to apparent power.

$$Power\ factor = \frac{working\ power}{apparent\ power} = \frac{kW}{kVA}$$

For example, if the working power is 400 kW and the apparent power is 500 kVA, the Power Factor would be 0.8 which is a relatively poor Power Factor. The closer the Power Factor is to 1, the better.

A poor Power Factor indicates your operations are not optimizing the usage of the electrical power being supplied to the premises. If your business is billed on a kVA demand tariff then your network charges will be higher than what they would be if you increased your Power Factor. If you're billed on a kW demand tariff then improving your Power Factor will not have a financial benefit. You can tell if you're billed on kVA or kW demand tariff by looking at the top right-hand corner of the first page of Energy Action's monthly reports, the "unit code' will show you how you are billed.

## 8.7 PARALLEL RLC CIRCUIT:



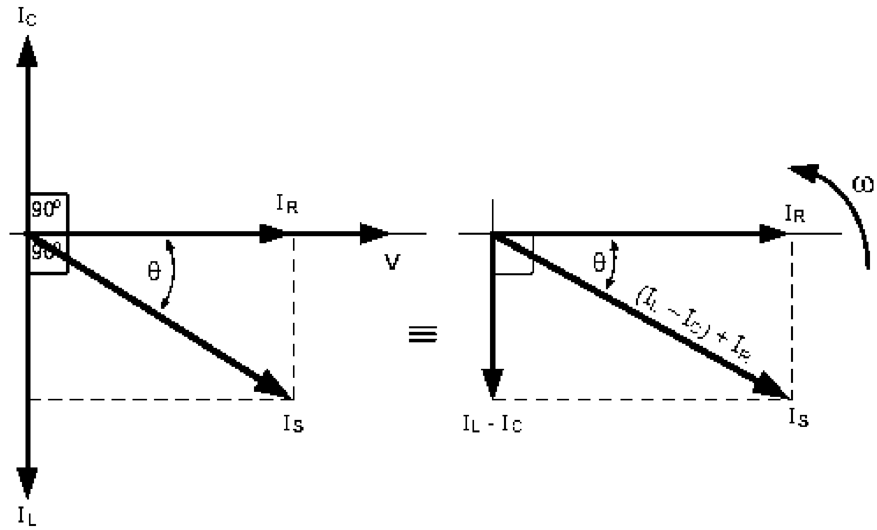

In the above parallel RLC circuit, we can see that the supply voltage, $V_S$ is common to all three components while the supply current $I_S$ consisting of three parts. The current flowing through the resistor $I_R$, the current flowing through the inductor $I_L$, and the current through the capacitor, is $I_C$.

But the current flowing through each branch and therefore each component will be different to each other and to the supply current, $I_S$. The total current drawn from the supply will not be the mathematical sum of the three individual branch currents but their vector sum.

Like the series RLC circuit, we can solve this circuit using the Phasor or vector method but this time the vector diagram will have the voltage as its reference with the three current vectors plotted with respect to the voltage. The Phasor diagram for a parallel RLC circuit is produced by combining together the three individual Phasor for each component and adding the currents vector sum.

Since the voltage across the circuit is common to all three circuit elements, we can use this as the reference vector with the three current vectors drawn relative to this at their corresponding angles. The resulting vector $I_S$ is obtained by adding together two of the vectors, $I_L$ and $I_C$ and then adding this sum to the remaining vector $I_R$. The resulting angle obtained between V and $I_S$ will be the circuits phase angle as shown below.

## PHASOR DIAGRAM FOR A PARALLEL RLC CIRCUIT:




We can see from the Phasor diagram on the right hand side above that the current vectors produce a rectangular triangle, comprising of hypotenuse $I_S$, horizontal axis $I_R$ and vertical axis $I_L - I_C$, this forms a Current Triangle and we can therefore use Pythagoras's theorem on this current triangle to mathematically obtain the magnitude of the branch currents along the x-axis and y-axis and then determine the total current $I_S$ of these components as shown above.

## CURRENT IN A PARALLEL RLC CIRCUIT:

$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

$$\therefore\ I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$\text{where:}\quad I_R = \frac{V}{R},\quad I_L = \frac{V}{X_L},\quad I_C = \frac{V}{X_C}$$

## IMPEDANCE OF A PARALLEL RLC CIRCUIT:

$$R = \frac{V}{I_R} \quad X_L = \frac{V}{I_L} \quad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

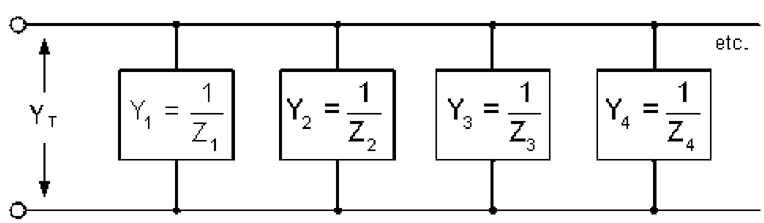
$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

You will notice that the final equation for a parallel RLC circuit produces complex impedances for each parallel branch as each element becomes the reciprocal of impedance, (1/Z) with the reciprocal of impedance being called Admittance.

In parallel AC circuits it is more convenient to use admittance, symbol (Y) to solve complex branch impedance's especially when two or more parallel branch impedances are involved. The total admittance of the circuit can simply be found by the addition of the parallel admittances. Then the total impedance, $Z_T$ of the circuit will therefore be $1/Y_T$ Siemens as shown.

## ADMITTANCE OF A PARALLEL RLC CIRCUIT:

$$\frac{1}{Z_T} = Y_T = Y_1 + Y_2 + Y_3 + Y_4 + \ldots\ldots\text{etc}$$




The new unit for admittance is the SIEMENS, abbreviated as S, (old unit mho's ℧, ohm's in reverse). Admittances are added together in parallel branches, whereas impedances are added together in series branches. But if we can have a reciprocal of impedance, we can also have a reciprocal of resistance and reactance as impedance consists of two components, R and X.

Then the reciprocal of resistance is called Conductance and the reciprocal of reactance is called Susceptance.

## CONDUCTANCE, ADMITTANCE AND SUSCEPTANCE:

The units for conductance, admittance and Susceptance are all the same namely Siemens (S), which can also be thought of as the reciprocal of Ohms or ohm$^{-1}$, but the symbol used for each element is different and in a pure component this is given as:

## ADMITTANCE (Y):

Admittance is the reciprocal of impedance, Z and is given the symbol Y. In AC circuits admittance is defined as the ease at which a circuit composed of resistances and reactance's allows current to flow when a voltage is applied taking into account the phase difference between the voltage and the current.

$$Y = \frac{1}{Z} \ [S]$$

The admittance of a parallel circuit is the ratio of Phasor current to Phasor voltage with the angle of the admittance being the negative to that of impedance.

## CONDUCTANCE (G):

Conductance is the reciprocal of resistance, R and is given the symbol G. Conductance is defined as the ease at which a resistor (or a set of resistors) allows current to flow when a voltage, either AC or DC is applied.

$$G = \frac{1}{R} \ [S]$$

## SUSCEPTANCE (B):

Susceptance is the reciprocal of reactance, X and is given the symbol B. In AC circuits Susceptance is defined as the ease at which a reactance (or a set of reactance) allows current to flow when a voltage is applied.

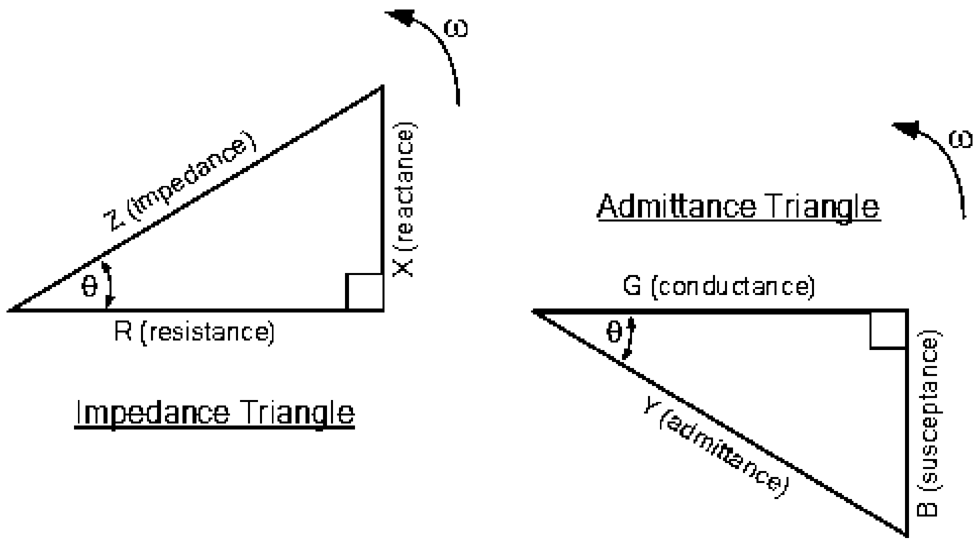
$$B_L = \frac{1}{X_L} \ [S]$$

$$B_C = \frac{1}{X_C} \ [S]$$

Susceptance has the opposite sign to reactance so capacitive Susceptance $B_C$ is positive, +ve in value and inductive Susceptance $B_L$ is negative, -ve in value.

In AC series circuits the opposition to current flow is impedance, Z which has two components, resistance R and reactance, X and from these two components we can construct an impedance triangle. Similarly, in a parallel RLC circuit, admittance, Y also has two components, conductance, G and Susceptance, B. This makes it possible to construct an admittance triangle that has a horizontal conductance axis, G and a vertical Susceptance axis, jB as shown below.

## ADMITTANCE TRIANGLE FOR A PARALLEL RLC CIRCUIT:




Now that we have an admittance triangle, we can use Pythagoras to calculate the magnitudes of all three sides as well as the phase angle as shown. From Pythagoras,

$$Y = \sqrt{G^2 + \left(B_L - B_C\right)^2}$$

$$\text{where:}\quad Y = \frac{1}{Z} \qquad G = \frac{1}{R}$$

$$B_L = \frac{1}{\omega L} \qquad B_C = \omega C$$

Then we can define both the admittance of the circuit and the impedance with respect to admittance as:

$$\text{Admittance:}\quad Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{(\omega L)} - (\omega C)\right)^2}$$

$$\text{Impedance:}\quad Z = \frac{1}{Y} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{(\omega L)} - (\omega C)\right)^2}}$$

Giving us a power factor angle of:

$$\cos\phi = \frac{G}{Y} \qquad \phi = \cos^{-1}\left(\frac{G}{Y}\right)$$

or

$$\tan\phi = \frac{B}{G} \qquad \phi = \tan^{-1}\left(\frac{B}{G}\right)$$

As the admittance, Y of a parallel RLC circuit is a complex quantity, the admittance corresponding to the general form of impedance Z = R + jX for series circuits will be written as Y = G - jB for parallel circuits where the real part G is the conductance and the imaginary part jB is the Susceptance. In polar form this will be given as:

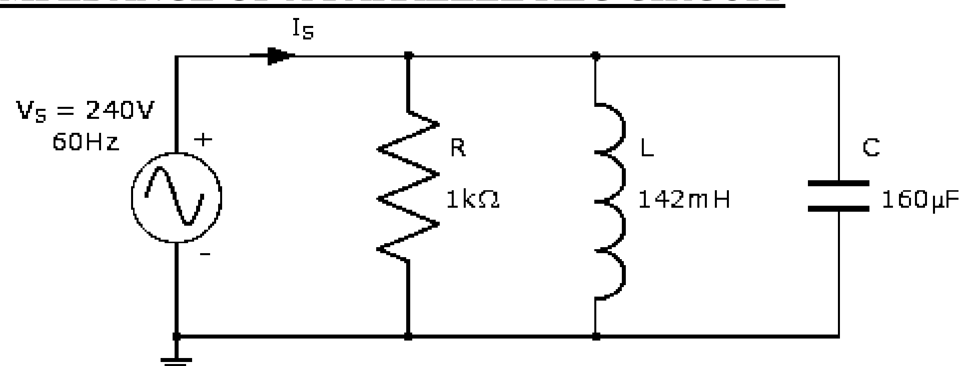
$$Y = G + jB = \sqrt{G^2 + B^2}\angle\tan^{-1}\frac{B}{G}$$

## 8.8 PARALLEL RLC CIRCUIT

## EXAMPLE NO.1

A 1kΩ resistor, a 142mH coil and a 160uF capacitor are all connected in parallel across a 240V, 60Hz supply. Calculate the impedance of the parallel RLC circuit and the current drawn from the supply.

## IMPEDANCE OF A PARALLEL RLC CIRCUIT




In an AC circuit, the resistor is unaffected by frequency therefore R = 1kΩ's

Inductive Reactance, ($X_L$):

$$X_L = \omega L = 2\pi f L = 2\pi.60.142\times10^{-3} = 53.54\Omega$$

Capacitive Reactance, ($X_C$):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.60.160\times10^{-6}} = 16.58\Omega$$

Impedance, ( Z ):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{53.54} - \frac{1}{16.58}\right)^2}}$$

$$Z = \frac{1}{\sqrt{1.0 \times 10^{-6} + 1.734 \times 10^{-3}}} = \frac{1}{0.0417} = 24.0\Omega$$

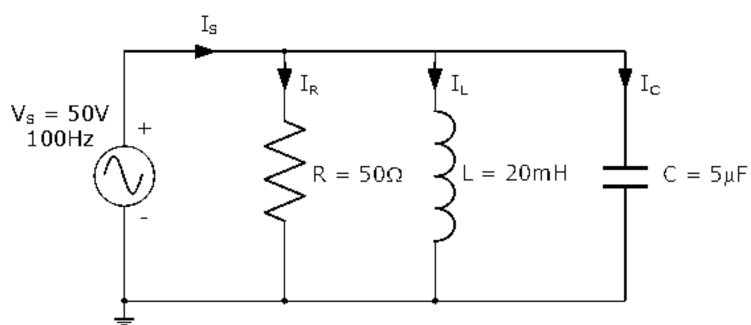
Supply Current, (Is):

$$I_S = \frac{V_S}{Z} = \frac{240}{24} = 10 \text{ Amperes}$$

## PARALLEL RLC CIRCUIT

## EXAMPLE NO.2

A 50Ω resistor, a 20mH coil and a 5μF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle. Also construct the current and admittance triangles representing the circuit.




1). Inductive Reactance, ($X_L$):

$$X_L = \omega L = 2\pi f L = 2\pi.100.0.02 = 12.6\Omega$$

2). Capacitive Reactance, ($X_C$):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.100.5 \times 10^{-6}} = 318.3\Omega$$

3). Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.3} - \frac{1}{12.6}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0058}} = \frac{1}{0.0788} = 12.7\Omega$$

4). Current through resistance, R ($I_R$):

$$I_R = \frac{V}{R} = \frac{50}{50} = 1.0\,(A)$$

5). Current through inductor, L ($I_L$):

$$I_L = \frac{V}{X_L} = \frac{50}{12.6} = 3.9\,(A)$$

6). Current through capacitor, C ($I_C$):

$$I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16\,(A)$$

7). Total supply current, ($I_S$):

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2} = \sqrt{1^2 + \left(3.9 - 0.16\right)^2} = 3.87\,(A)$$

8). Conductance, (G):

$$G = \frac{1}{R} = \frac{1}{50} = 0.02S \text{ or } 20mS$$

9). Inductive Susceptance, ($B_L$):

$$B_L = \frac{1}{X_L} = \frac{1}{12.6} = 0.08S \text{ or } 80mS$$

10). Capacitive Susceptance, ($B_C$):

$$B_C = \frac{1}{X_C} = \frac{1}{318.3} = 0.003S \text{ or } 3mS$$

11). Admittance, (Y):

$$Y = \frac{1}{Z} = \frac{1}{12.7} = 0.078S \text{ or } 78mS$$

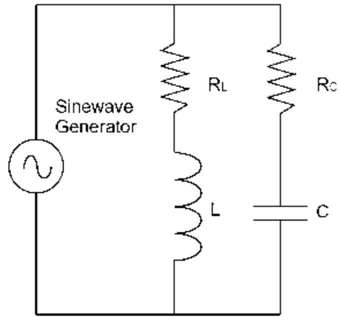
12). Phase Angle, ($\varphi$) between the resultant current and the supply voltage:

$$\cos\phi = \frac{G}{Y} = \frac{20mS}{78mS} = 0.256$$

$$\phi = \cos^{-1} 0.256 = 75.3^0 \quad (\text{lag})$$

## PARALLEL RLC CIRCUIT

## EXAMPLE NO.3




In the above circuit in 1st branch 50Ω resistor and 20mH inductance are connected in series, while in 2nd branch 50μF capacitor and 20Ω resistance is connected in series. If the both branches are connected in parallel across a 100V, 50Hz supply. Calculate the total current drawn from the supply.

**Note: There are three methods to solve parallel circuit, above quoted example has been solved by all three methods, students can adopt any one method. Examiners are requested not to make compulsory a particular method.**

## SOLUTION WITH ADMITTANCE METHOD:

**Branch # 01**

$R_L = 50\ \Omega$, $L = 20\ mH = 0.02\ H$, $V_T = 100V$, $F = 50Hz$

$X_L = 2\ \pi\ F\ L = 2 \times 3.142 \times 50 \times 0.02 = 6.284\ \Omega$

$Z_1 = \sqrt{R_L^{\ 2} + X_L^{\ 2}} = \sqrt{50^2 + 6.284^2} = 50.39\ \Omega$

$G_1 = \frac{R_L}{Z_1^2} = \frac{50}{50.39^2} = 0.0197\Omega$

$B_1 = B_L = \frac{X_L}{Z_1^2} = \frac{6.284}{50.39^2} = 0.0025\ \Omega$ (Lagging)

**Branch # 02**

$R_C = 20\ \Omega$, $C = 50\mu F$, $V_T = 100V$, $F = 50Hz$

$X_C = \frac{10^6}{2\ \pi\ F\ C} = \frac{10^6}{2 \times 3.142 \times 50 \times 50} = 63.654\ \Omega$

$Z_2 = \sqrt{R_C^{\ 2} + X_C^{\ 2}} = \sqrt{20^2 + 63.654^2} = 66.72\ \Omega$

$G_2 = \frac{R_C}{Z_2^2} = \frac{20}{66.72^2} = 0.0045\Omega$

$B_2 = B_C = -\frac{X_C}{Z_2^2} = \frac{63.654}{66.72^2} = -\ 0.0143\ \Omega$ (Leading)

$G_T = G_1 + G_2 = 0.0197 + 0.0045 = 0.0242\ \Omega$

$B_T = B_1 + B_2 = 0.0025 + (-\ 0.0143) = -\ 0.0118\ \Omega$ (Leading)

$Y_T = \sqrt{G_T{}^2 + B_T{}^2} = \sqrt{0.0242^2 + (-0.0118)^2} = 0.0279\ \Omega$

$I_T = V_T \times Y_T = 100 \times 0.0279 = 2.79$ A (Leading)

## SOLUTION WITH VECTOR METHOD:

**Branch # 01**

$R_L = 50\ \Omega$, L = 20 mH = 0.02 H, $V_T = 100$V, F= 50Hz

$X_L = 2\pi F L = 2 \times 3.142 \times 50 \times 0.02 = 6.284\ \Omega$

$Z_1 = \sqrt{R_L{}^2 + X_L{}^2} = \sqrt{50^2 + 6.284^2} = 50.39\ \Omega$

$I_1 = \frac{V_T}{Z_1} = \frac{100}{50.39} = 1.9845$ A

$\cos \Phi_1 = \frac{R_L}{Z_1} = \frac{50}{50.39} = 0.9922$ lag

$\sin \Phi_1 = \frac{X_L}{Z_1} = \frac{6.284}{50.39} = 0.0125$

Active component $= I_1 \cos \Phi_1 = 1.9845 \times 0.9922 = 1.969$ A

Reactive component $= I_1 \sin \Phi_1 = 1.9845 \times 0.0125 = 0.0248$ A

**Branch # 02**

$R_C = 20\ \Omega$, C = 50µF, $V_T = 100$V, F= 50Hz

$X_C = \frac{10^6}{2\pi F C} = \frac{10^6}{2 \times 3.142 \times 50 \times 50} = 63.654\ \Omega$

$Z_2 = \sqrt{R_C{}^2 + X_C{}^2} = \sqrt{20^2 + 63.654^2} = 66.72\ \Omega$

$I_2 = \frac{V_T}{Z_2} = \frac{100}{66.72} = 1.4988$ A

$\cos \Phi_2 = \frac{R_C}{Z_2} = \frac{20}{66.72} = 0.2997$ lead

$\sin \Phi_2 = \frac{X_C}{Z_2} = \frac{63.654}{66.72} = 0.954$

Active component $= I_2 \cos \Phi_2 = 1.4988 \times 0.2997 = 0.4494$ A

Reactive component $= -I_2 \sin \Phi_2 = -1.4988 \times 0.954 = -1.4299$ A

Total active component $= I_T \cos \Phi_T = I_1 \cos \Phi_1 + I_2 \cos \Phi_2$
$= 1.969 + 0.4494 = 2.4184$ A

Total reactive component $= I_T \sin \Phi_T = I_1 \sin \Phi_1 + I_2 \sin \Phi_2$
$= 0.0248 + (-1.4299) = -1.4051$ A

$I_T = \sqrt{I_T \cos \Phi_T{}^2 + I_T \sin \Phi_T{}^2} = \sqrt{2.4184^2 + (-1.4051)^2}$
$= 2.796$ A (Leading)

## SOLUTION WITH J METHOD:

**Branch # 01**

$R_L = 50\ \Omega$, $L = 20\text{ mH} = 0.02\text{ H}$, $V_T = 100V\angle 0°$, $F = 50Hz$

$X_L = 2\pi F L = 2 \times 3.142 \times 50 \times 0.02 = 6.284\ \Omega$

$Z = R_L + JX_L = 50 + J6.284 = 50.39 \angle 7.16°\ \Omega$

$$I_1 = \frac{V_T}{Z_1} = \frac{100 \angle 0°}{50.39 \angle 7.16°} = 1.9845 \angle -7.16°\text{ A} = (1.969 - J0.0248)\text{ A}$$

**Branch # 02**

$R_C = 20\ \Omega$, $C = 50\mu F$, $V_T = 100V\angle 0°$, $F = 50Hz$

$$X_C = \frac{10^6}{2\pi F C} = \frac{10^6}{2 \times 3.142 \times 50 \times 50} = 63.654\ \Omega$$

$Z_2 = R_L - JX_C = 20 - J63.654 = 66.72 \angle -72.56°\ \Omega$

$$I_2 = \frac{V_T}{Z_2} = \frac{100 \angle 0°}{50.39 \angle -72.56°} = 1.9845 \angle 72.56°\text{ A}$$

$= (0.2997 + J1.4299)\text{ A}$

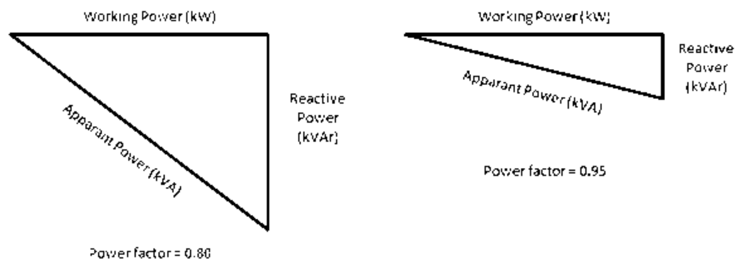
$I_T = I_1 + I_2 = (1.969 - J0.0248) + (0.2997 + J1.4299)$

$= (2.4184 + J1.4051)\text{ A}$

$I_T = 2.796 \angle 30.15°\text{ A (Leading)}$

## 8.9 POWER FACTOR IMPROVEMENT:

The simplest way to improve Power Factor is to install a Power Factor Correction unit at your site. Power factor correction units consist of capacitors which act as reactive current generators. By providing the reactive power, they reduce the total amount of power you must draw from the network. For the same working power (kW) you can reduce the reactive power (kVAr) and the apparent power (kVA) as shown below:




## METHODS FOR POWER FACTOR IMPROVEMENT

The following devices and equipment's are used for Power Factor Improvement.

1. Static Capacitor
2. Synchronous Condenser
3. Phase Advancer

Power factor improvement with capacitor is being discussed in detail.

## 1. STATIC CAPACITOR:

We know that most of the industries and power system loads are inductive that take lagging current which decrease the system power factor. For Power factor improvement purpose, Static capacitors are connected in parallel with those devices which work on low power factor. These static capacitors provide leading current which neutralize (totally or approximately) the lagging inductive component of load current (i.e. leading component neutralize or eliminate the lagging component of load current) thus power factor of the load circuit is improved. These capacitors are installed in Vicinity of large inductive load e.g. Induction motors and transformers etc., and improve the load circuit power factor to improve the system or devises efficiency.

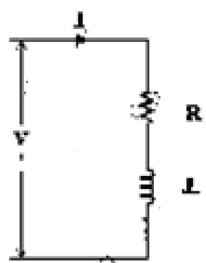



Fig 1

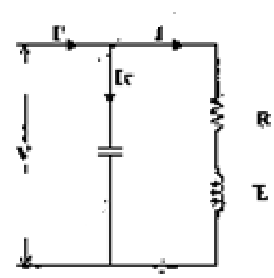



Fig 2


Fig 3

**Suppose**, here is a single-phase inductive load which is taking lagging current (I) and the load power factor is Cos θ as shown in fig-1. In fig-2, a Capacitor (C) has been connected in parallel with load. Now a current (Ic) is flowing through Capacitor which lead 90° from the supply voltage (Note that Capacitor provides leading Current i.e., In a pure capacitive circuit, Current leading 90° from the supply Voltage, in other words, Voltage are 90° lagging from Current). The load current is (I). The Vectors combination of (I) and (Ic) is (I') which is lagging from voltage at $\theta_2$ as shown in fig 3.

It can be seen from fig 3 that angle of $\theta_2 < \theta_1$ i.e. angle of $\theta_2$ is less than from angle of $\theta_2$. Therefore, $\text{Cos}\theta_2$ is less than from $\text{Cos}\theta_1$ ($\text{Cos}\theta_2 > \text{Cos}\theta_1$). Hence the load power factor is improved by capacitor.

Also note that after the power factor improvement, the circuit current would be less than from the low power factor circuit current. Also, before and after the power factor improvement, the active component of current would be same in that circuit because capacitor eliminates only the re-active component of current. Also, the Active power (in Watts) would be same after and before power factor improvement.

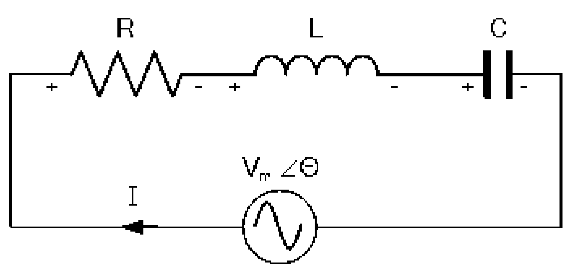
## ADVANTAGES:

- Capacitor bank offers several advantages over other methods of power factor improvement.
- Losses are low in static capacitors
- There is no moving part, therefore need low maintenance
- It can work in normal air conditions (i.e. ordinary atmospheric conditions)
- Do not require a foundation for installation
- They are lightweight so it is can be easy to installed

## DISADVANTAGES:

- The age of static capacitor bank is less (8 – 10 years)
- With changing load, we have to ON or OFF the capacitor bank, which causes switching surges on the system
- If the rated voltage increases, then it causes damage it
- Once the capacitors spoiled, then repairing is costly

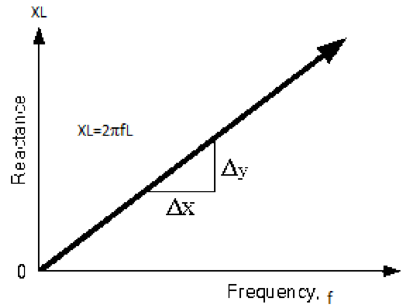
## 8.10 SERIES OR VOLTAGE RESONANCE:




Consider a circuit containing R, L and C in series, then

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

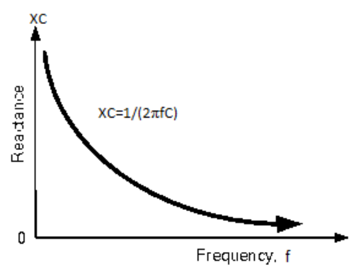
However, as the frequency approaches zero or DC, the inductors reactance would decrease to zero, causing the opposite effect acting like a short circuit. This means then that inductive reactance is “Proportional” to frequency and is small at low frequencies and high at higher frequencies and this demonstrated in the following curve:




The graph of inductive reactance against frequency is a straight-line linear curve. The inductive reactance value of an inductor increases linearly as the frequency across it increases. Therefore, inductive reactance is positive and is directly proportional to frequency ($X_L \propto f$)

The same is also true for the capacitive reactance formula above but in reverse. If either the Frequency or the **Capacitance** is increased the overall capacitive reactance would decrease. As the frequency approaches infinity, the capacitors reactance would reduce to zero causing the circuit element to act like a perfect conductor of 0Ω's.

But as the frequency approaches zero or DC level, the capacitors reactance would rapidly increase up to infinity causing it to act like very large resistance acting like an open circuit condition. This means then that capacitive reactance is "Inversely proportional" to frequency for any given value of capacitance and this shown below:



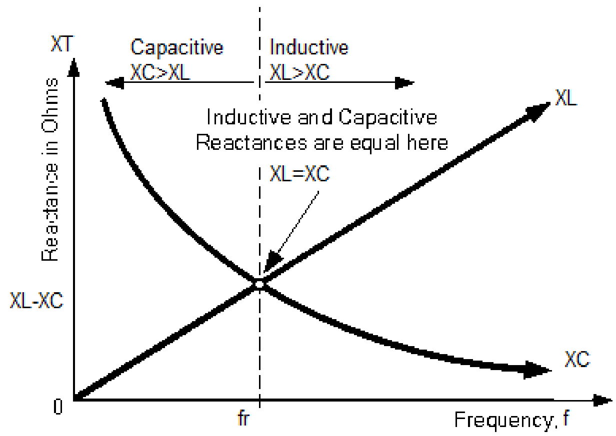

The graph of capacitive reactance against frequency is a hyperbolic curve. The Reactance value of a capacitor has a very high value at low frequencies but quickly decreases as the frequency across it increases. Therefore, capacitive reactance is negative and is inversely proportional to frequency ($X_C \propto f^{-1}$)

In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, $X_L = X_C$. The point at which this occurs is called the Resonant Frequency point, ($o_r$) of the circuit, and as we are analyzing a series RLC circuit this resonance frequency produces a Series Resonance**.**

Series Resonance circuits are one of the most important circuits used electrical and electronic circuits. They can be found in various forms such as in AC mains filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels.

## SERIES RESONANCE FREQUENCY

At a higher frequency $X_L$ is high and at a low frequency $X_C$ is high. Then there must be a frequency point were the value of $X_L$ is the same as the value of $X_C$ and there is. If we now place the curve for inductive reactance on top of the curve for capacitive reactance so that both curves are on the same axes, the point of intersection will give us the series resonance frequency point, ($f_r$ or $\omega_r$) as shown below.




Where: $f_r$ is in Hertz, L is in Henry and C is in Farad.

Electrical resonance occurs in an AC circuit when the two reactance's which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is where the two reactance curves cross each other. In a series resonant circuit, the resonant frequency, $f_r$ point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

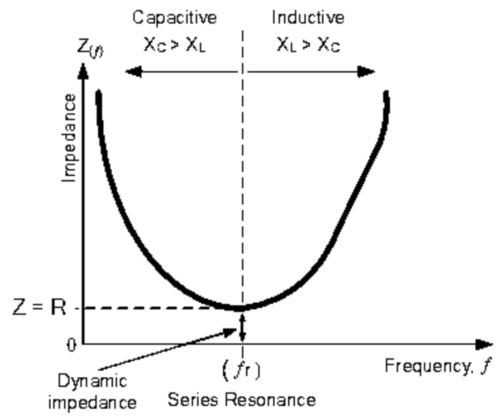
$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz)} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \text{ (rads)}$$

## IMPEDANCE IN A SERIES RESONANCE CIRCUIT

At resonance, the two reactance's cancel each other out thereby making a series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R. In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit becomes purely **"**REAL**"**, that is no imaginary impedances exist. This is because at resonance they are cancelled out. So, the total impedance of the series circuit becomes just the value of the resistance and therefore: Z = R.

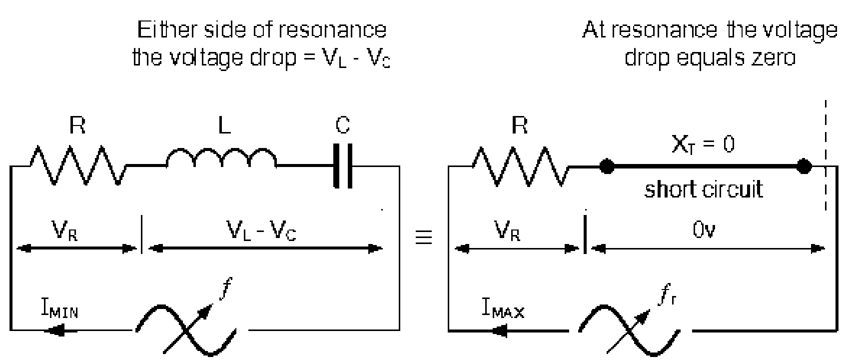
Then at resonance the impedance of the series circuit is at its minimum value and equal only to the resistance, R of the circuit. The circuit impedance at resonance is called the "dynamic impedance" of the circuit and depending upon the frequency, $X_C$ (typically at high frequencies) or $X_L$ (typically at low frequencies) will dominate either side of resonance as shown below.




Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of $X_L$. You may also note that if the circuit's impedance is at its minimum at resonance then consequently, the circuit's admittance must be at its maximum and one of the characteristics of a series resonance circuit is that admittance is very high. But this can be a bad thing because a very low value of resistance at resonance means that the circuits current may be dangerously high.

## VOLTAGE IN SERIES RESONANCE CIRCUIT:

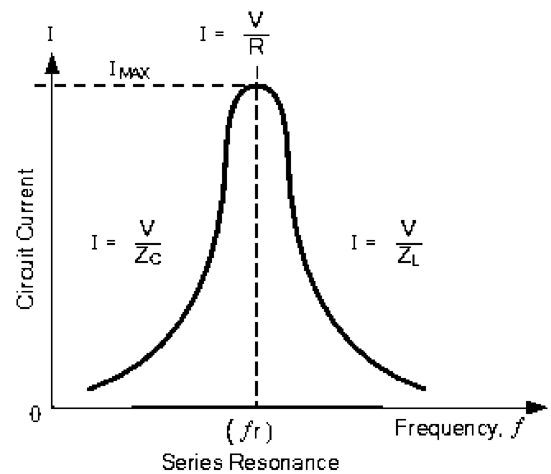
In RLC circuits the voltage across a series combination is the Phasor sum of $V_R$, $V_L$ and $V_C$. Then if at resonance the two reactance's are equal and cancelling, the two voltages representing $V_L$ and $V_C$ must also be opposite and equal in value thereby cancelling each other out because with pure components the Phasor voltages are drawn at $+90^o$ and $-90^o$ respectively. Then in a series resonance circuit $V_L = -V_C$ therefore, $V = V_R$.



## CURRENT IN SERIES RESONANCE CIRCUIT:

Since the current flowing through a series resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its minimum value, (Z=R). Therefore, the circuit current at this frequency will be at its maximum value of V/R as shown below.

The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as $f$ becomes infinite.




The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition, they cancel each other out.

As a series resonance circuit only functions on resonant frequency, this type of circuit is also known as an Acceptor Circuit because at resonance, the impedance of the circuit is at its minimum so easily accepts the current whose frequency is equal to its resonant frequency. The effect of resonance in a series circuit is also called “voltage resonance”.

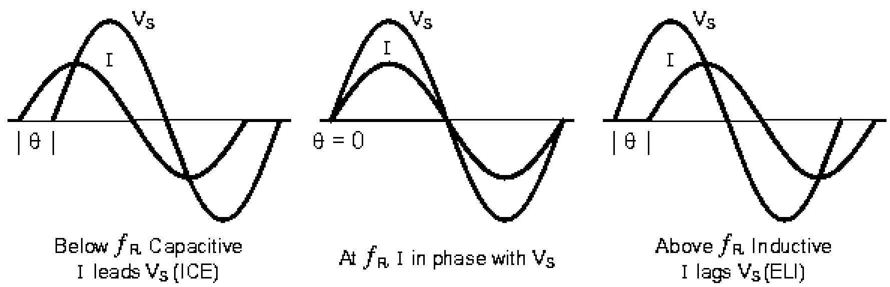
at $\omega_r$ $\quad Z_T = \text{min}, \quad I_S = \text{max}$

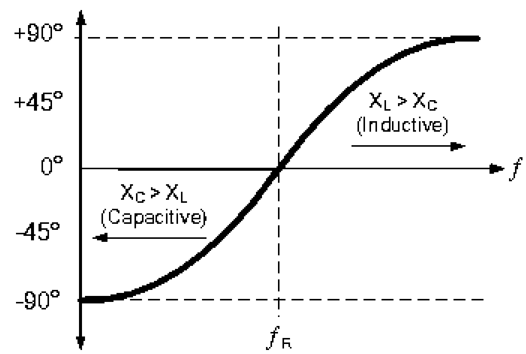
$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

## PHASE ANGLE OF A SERIES RESONANCE CIRCUIT:

Notice also, that the phase angle is positive for frequencies above $f_r$ and negative for frequencies below $f_r$ and this can be proven by,

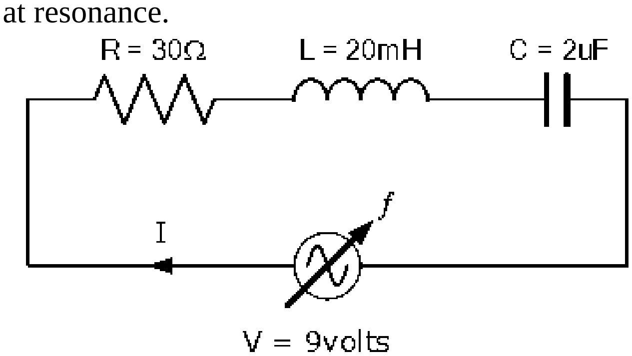
$$\tan\phi = \frac{X_L - X_C}{R} = 0^0 \quad (\text{all real})$$




## SERIES RESONANCE
## EXAMPLE NO.1

A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies. Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance.




Resonant Frequency, $f_r$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times2\times10^{-6}}} = 796\text{Hz}$$

Circuit Current at Resonance, $I_m$

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A or } 300\text{mA}$$

Inductive Reactance at Resonance, $X_L$

$$X_L = 2\pi f L = 2\pi\times796\times0.02 = 100\Omega$$

Voltages across the inductor and the capacitor, $V_L$, $V_C$

$$V_L = V_C$$
$$V_L = I\times X_L = 300\text{mA}\times100\Omega$$
$$V_L = 30\text{volts}$$

## SERIES RESONANCE

## EXAMPLE NO.2

A series circuit consists of a resistance of 4Ω, an inductance of 500mH and a variable capacitance connected across a 100V, 50Hz supply. Calculate the capacitance require to give series resonance and the voltages generated across both the inductor and the capacitor.
Resonant Frequency, $f_r$

$$X_L = 2\pi f L = 2\pi\times50\times0.5 = 157.1\Omega$$

$$\text{at resonance: } X_C = X_L = 157.1\Omega$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi.50.157.1} = 20.3\mu\text{F}$$

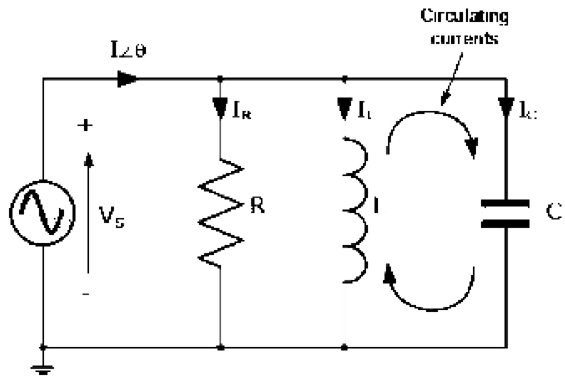
Voltages across the inductor and the capacitor, $V_L$, $V_C$

$$I_S = \frac{V}{R} = \frac{100}{4} = 25\text{Amps}$$

$$\text{at resonance: } V_L = V_C$$

$$V_L = I\times X_L = 25\times157.1$$
$$\therefore V_L = 3{,}927.5\text{volts}$$

## PARALLEL RESONANCE RLC CIRCUIT:




Let us define what we already know about parallel RLC circuits.

$$\text{Admittance, } Y = \frac{1}{Z} = \sqrt{G^2 - B^2}$$

$$\text{Conductance, } G = \frac{1}{R}$$

$$\text{Inductive Susceptance, } B_L = \frac{1}{2\pi f L}$$

$$\text{Capacitive Susceptance, } B_C = 2\pi f C$$

A parallel circuit containing a resistance, R, an inductance, L and a capacitance, C will produce a parallel resonance (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage. At resonance there will be a large circulating current between the inductor and the capacitor due to the energy of the oscillations.

A parallel resonant circuit stores the circuit energy in the magnetic field of the inductor and the electric field of the capacitor. This energy is constantly being transferred back and forth between the inductor and the capacitor which results in zero current and energy being drawn from the supply. This is because the corresponding instantaneous values of $I_L$ and $I_C$ will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in $I_R$.

In the solution of AC parallel resonance circuits, we know that the supply voltage is common for all branches, so this can be taken as our reference vector. Each parallel branch must be treated separately as with series circuits so that the total supply current taken by the parallel circuit is the vector addition of the individual branch currents. Then there are two methods available to us in the analysis of parallel resonance circuits. We can calculate the current in each branch and then add together or calculate the admittance of each branch to find the total current.

We know from the previous series resonance tutorial that resonance takes place when $V_L = -V_C$ and this situation occurs when the two reactance's are equal, $X_L = X_C$. The admittance of a parallel circuit is given as:

$$Y = G + B_L + B_C$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

or

$$Y = \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$$

Resonance occurs when $X_L = X_C$ and the imaginary parts of Y become zero. Then:

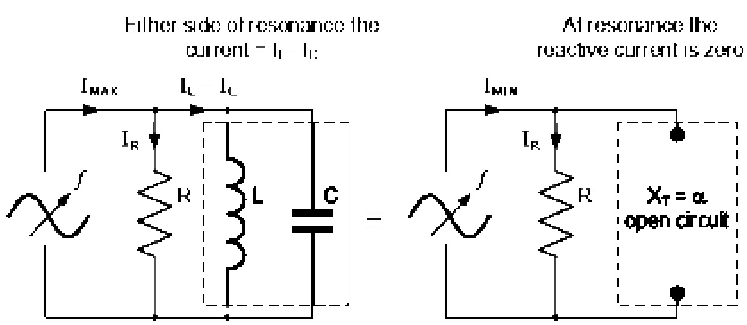
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \,(\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \,(\mathrm{rads})$$

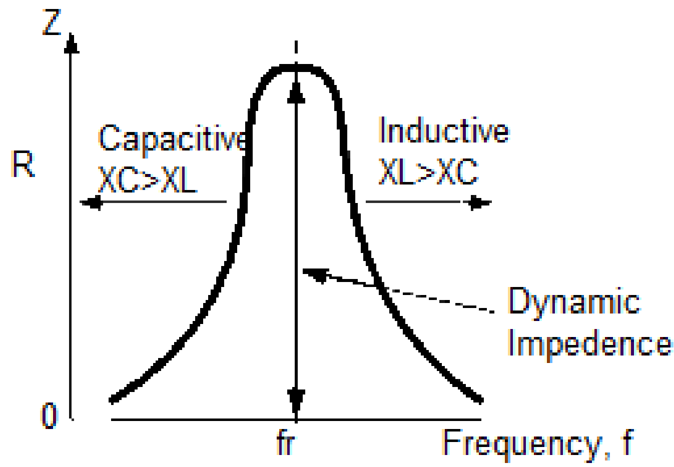
Notice that at resonance the parallel circuit produces the same equation as for the series resonance circuit. Therefore, it makes no difference if the inductor or capacitors are connected in parallel or series. Also, at resonance the parallel LC tank circuit acts like an open circuit with the circuit current being determined by the resistor, R only. So, the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and Z = R as shown.



At resonance, the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit. Also, at resonance, as the impedance of the circuit is now that of resistance only, the total circuit current, I will be "in-phase" with the supply voltage, $V_S$.

We can change the circuit's frequency response by changing the value of this resistance. Changing the value of R affects the amount of current that flows through the circuit at resonance, if both L and C remain constant. Then the impedance of the circuit at resonance $Z = R_{MAX}$ is called the "dynamic impedance" of the circuit.

## IMPEDANCE IN A PARALLEL RESONANCE CIRCUIT:



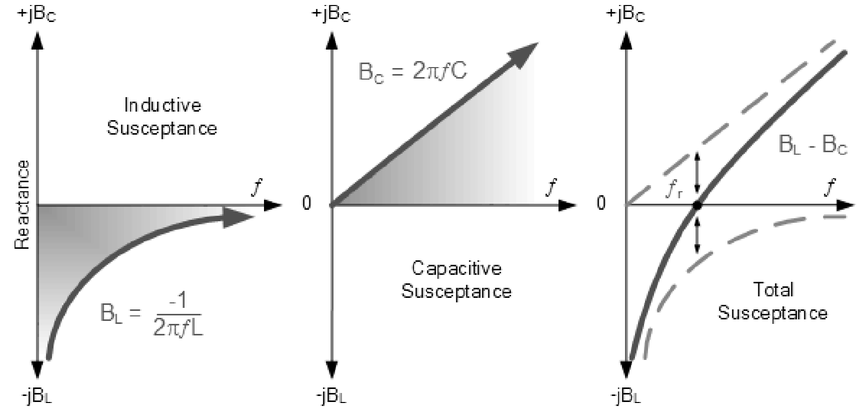

Parallel Resonance

Note that if the parallel circuit's impedance is at its maximum at resonance then consequently, the circuit's admittance must be at its minimum and one of the characteristics of a parallel resonance circuit is that admittance is very low limiting the circuits current. Unlike the series resonance circuit, the resistor in a parallel resonance circuit has a damping effect on the circuit's bandwidth making the circuit less selective.

Also, since the circuit current is constant for any value of impedance, Z, the voltage across a parallel resonance circuit will have the same shape as the total impedance and for a parallel circuit the voltage waveform is generally taken from across the capacitor.

We now know that at the resonant frequency, $f_r$ the admittance of the circuit is at its minimum and is equal to the conductance, G given by 1/R because in a parallel resonance circuit the imaginary part of admittance, i.e. the Susceptance, B is zero because $B_L = B_C$ as shown.

## SUSCEPTANCE AT RESONANCE




From above, the INDUCTIVE SUSCEPTANCE, $B_L$ is inversely proportional to the frequency as represented by the hyperbolic curve. The CAPACITIVE SUSCEPTANCE, $B_C$ is directly proportional to the frequency and is therefore represented by a straight line. The final curve shows the plot of total Susceptance of the parallel resonance circuit versus the frequency and is the difference between the two susceptance.

Then we can see that at the resonant frequency point where it crosses the horizontal axis the total circuit susceptance is zero. Below the resonant frequency point, the inductive susceptance dominates the circuit producing a "lagging" power factor, whereas above the resonant frequency point the capacitive susceptance dominates producing a "leading" power factor. So, at resonant frequency, the circuits current must be "in-phase" with the applied voltage as there effectively there is only the resistance in the circuit so the power factor becomes one or unity, ($\theta = 0^o$).

## CURRENT IN A PARALLEL RESONANCE CIRCUIT:

As the total susceptance is zero at the resonant frequency, the admittance is at its minimum and is equal to the conductance, G. Therefore, at resonance the current flowing through the circuit must also be at its minimum as the inductive and capacitive branch currents are equal ($I_L = I_C$) and are $180^o$ out of phase.

We remember that the total current flowing in a parallel RLC circuit is equal to the vector sum of the individual branch currents and for a given frequency is calculated as:

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V.2\pi f C$$

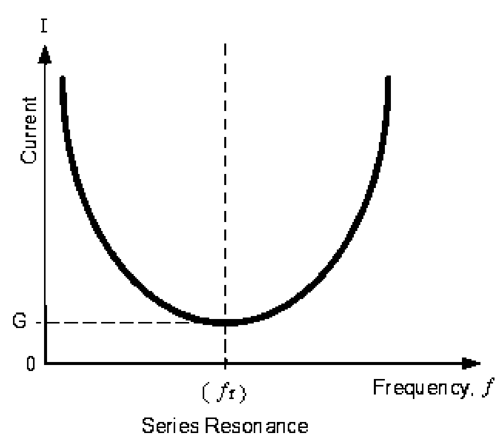
$$\text{Therefore, } I_T = \text{vector sum of } (I_R + I_L + I_C)$$

$$I_T = \sqrt{I_R^2 + (I_L + I_C)}$$

At resonance, currents $I_L$ and $I_L$ are equal and cancelling giving a net reactive current equal to zero. Then at resonance the above equation becomes.

$$I_T = \sqrt{I_R^2 + 0^2} = I_R$$

Since the current flowing through a parallel resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its maximum value, (=R). Therefore, the circuit current at this frequency will be at its minimum value of V/R and the graph of current against frequency for a parallel resonance circuit is given as.




The frequency response curve of a parallel resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at its maximum value, reaches its minimum value at the resonance frequency when $I_{MIN} = I_R$ and then increases again to maximum as $f$ becomes infinite.

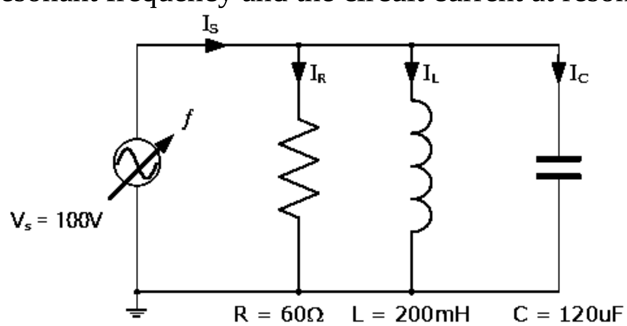
The result of this is that the magnitude of the current flowing through the inductor, L and the capacitor, C tank circuit can become many times larger than the supply current, even at resonance but as they are equal and at opposition ( $180^o$ out-of-phase ) they effectively cancel each other out.

As a parallel resonance circuit only functions on resonant frequency, this type of circuit is also known as a Rejecter Circuit because at resonance, the impedance of the circuit is at its maximum thereby suppressing or rejecting the current whose frequency is equal to its resonant frequency. The effect of resonance in a parallel circuit is also called "current resonance".

The calculations and graphs used above for defining a parallel resonance circuit are similar to those we used for a series circuit. However, the characteristics and graphs drawn for a parallel circuit are exactly opposite to that of series circuits with the parallel circuit's maximum and minimum impedance, current and magnification being reversed. Which is why a parallel resonance circuit is also called an Anti-resonance circuit.

## PARALLEL RESONANCE EXAMPLE:

A parallel resonance network consisting of a resistor of 60Ω, a capacitor of 120uF and an inductor of 200mH is connected across a sinusoidal supply voltage which has a constant output of 100 volts at all frequencies. Calculate the resonant frequency and the circuit current at resonance.




Resonant Frequency, $f_r$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2.120.10^{-6}}} = 32.5\text{Hz}$$

Inductive Reactance at Resonance, $X_L$

$$X_L = 2\pi f L = 2\pi.32.5.0.2 = 40.8\Omega$$

Circuit Current at Resonance, $I_T$

At resonance the dynamic impedance of the circuit is equal to R

$$I_T = I_R = \frac{V}{R} = \frac{100}{60} = 1.67\text{A}$$

Note that the current at resonance (the resistive current) is only 1.67 amps, while the current flowing around the LC tank circuit is larger at 2.45 amps. We can check this value by calculating the current flowing through the inductor (or capacitor) at resonance.

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L} = \frac{100}{2\pi.32.5.0.2} = 2.45\text{A}$$

## LINEAR LOAD

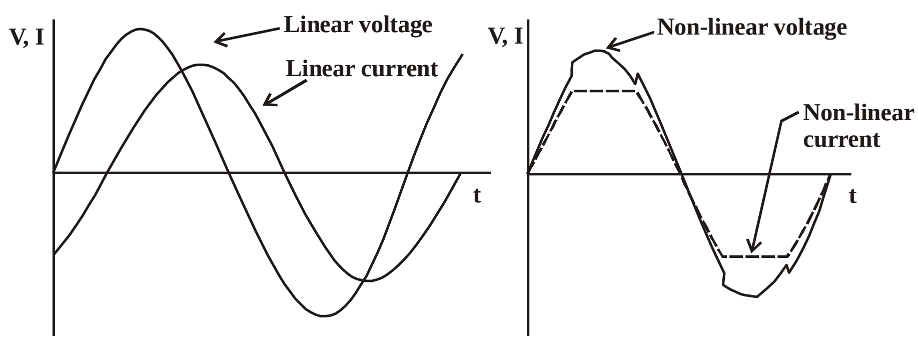
If the current through a load varies proportionally to the voltage across it and follows the voltage, the load is linear with a linear load, relationship between the voltage & current waves are sinusoidal and the current at any time is proportional to the voltage (Ohm's law).

**Examples:** Resistor, Inductor, Capacitors, Transformer, Motor etc.

## NON-LINEAR LOAD

With a non-linear load, the current waveform is not proportional to the voltage waveform. (Figure (b)). Non-linear load draw in currents in abrupt short pulses. These pulses distort the current waveforms which generates harmonics.

**Examples:** Electronic devices, Diodes, Thyristors etc.




**(a) Voltage & current of a linear load (b) Voltage & current of a non-linear load**

## HARMONICS

Harmonics are unwanted electrical waves that flow in the line along with the generated power. Those waves whose frequency is not equal to fundamental frequency are called as "Harmonics".

In an electric power system, a harmonic is a voltage or current at a multiple of the fundamental frequency of the system.

Harmonics are multiples of the fundamental frequency and can therefore be expressed as 2f, 3f, 4f, etc. as shown in figure (a). The fundamental waveform can also be called 1st harmonic waveform. 2nd harmonic has a frequency twice that of fundamental; the 3rd harmonic has 3 times of the fundamental frequency etc. as shown in figure (a).

If the fundamental frequency is given as $E_1 = V_m\ (3\square ft)$ or $V_m(\square t)$ then the 2nd harmonic, $E_2 = V_{m-2}\ (2\square t)$ and the 3rd harmonic, $E_3 = V_{m-3}\ (3\square t)$

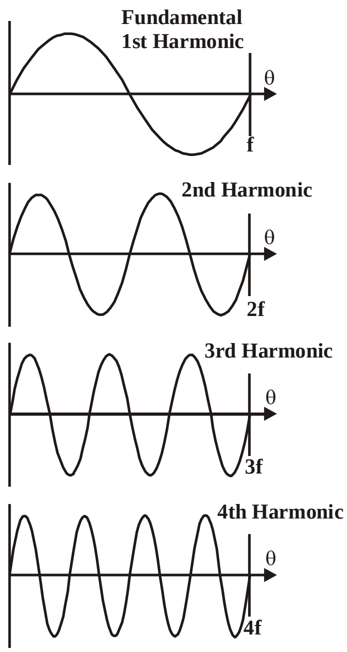



Fig. (a) Harmonic Waveform

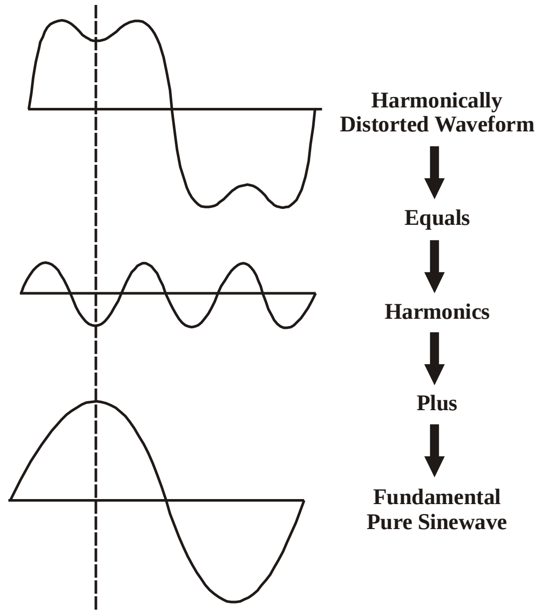



Fig (b)

Harmonics are produced by the action of non-linear loads. These harmonics create disturbance which affect the wave shape as given in figure (b). Harmonics increase power system heat losses & power bills to end-users.

Before explanation of true power factor, we have to understand displacement power factor & distortion power factor.

## Displacement Power Factor

The linear devices draw sinusoidal current waveform sinusoidal supply source & these devices do not distort the current waveform. The current drawn by these devices is sinusoidal but the current waveform is not in phase with the voltage.

The cosine of the phase angle between voltage & current is called the Displacement Power Factor (DPF) or the Fundamental Power Factor.

DPF = Cos $\phi$ (Where $\phi$ is phase angle between V & I)

## Distortion power factor

When the load connected to systems are non-linear, the current is not sinusoidal & it distorts the fundamental current waveform. The power factor with non-linear loads is known as Distortion Power Factor.

## True power factor

The true power factor is the product of the displacement power factor & distortion power factor.

True power factor = Displacement p.f. × Distortion p.f

# EXERCISE # 08

## PART-A

**Chose the correct answer**

1. In pure resistive circuit, the current & voltage are:
   (a) On 180º (b) On 90º
   (c) Out of phase (d) In phase
2. Phase angle of pure resistive circuit is:
   (a) 0º (b) 30º (c) 60º (d) 90º
3. Power factor of pure resistive circuit is:
   (a) 0 (b) 0.5 (c) 0.8 (d) Unity
4. Formula to determine current in pure resistive circuit is:
   (a) $\frac{V}{R}$ (b) $\frac{W}{V}$ (c) $\frac{W}{V.\cos\theta}$ (d) All
5. Current lags with voltage in pure inductive circuit:
   (a) 0º (b) 0 - 90º (c) 90º (d) 120°
6. Phase angle between V & I in pure inductive circuit:
   (a) 30º (b) 60º (c) 90º (d) In phase
7. In pure inductive circuit, current ___________ with voltage:
   (a) Leads (b) Lags (c) In phase (d) None
8. Power factor of pure inductive circuit is:
   (a) 0 (b) 0.5 (c) 0.707 (d) Unity
9. In pure inductive circuit, the formula to find the power is:
   (a) $P = VI$ (b) $P = I^2R$
   (c) $P = V^2 / R$ (d) $P = VI \cos\theta$
10. Power consumed in pure inductive circuit is:
    (a) 0 (b) Low
    (c) Medium (d) Maximum
11. By increasing in pure inductive circuit, XL is:
    (a) Increased (b) Decreased
    (c) Constant (d) Becomes zero
12. The unit of XL is:
    (a) Henry (b) Henry/m
    (c) Ohm/m (d) Ohm
13. Formula to determine the current in pure inductive circuit is:
    (a) $I = V/R$ (b) $I = V/X_L$
    (c) $I = V/Z$ (d) All
14. Formula to determine voltage in pure inductive circuit is:
    (a) $V = IR$ (b) $V = 1.X_L$
    (c) $V = IZ$ (d) All
15. In pure capacitive circuit, the current & voltage are:
    (a) On 0º (b) 0 - 90°
    (c) On 90º (d) 120°

16. Current leads with voltage in pure capacitive circuit:
    (a) Leads (b) Lags
    (c) In phase (d) None
17. Power factor of pure capacitive circuit is:
    (a) 0 (b) 0.707 (c) 0.636 (d) Unity
18. Formula to determine the formula of capacitive circuit is:
    (a) VI (b) $I^2R$ (c) $V^2/R$ (d) VI cos θ
19. Power consumed in pure inductive circuit is:
    (a) Zero (b) Very low
    (c) Medium (d) High
20. In capacitive circuit, with increasing the frequency, $X_C$ is:
    (a) Increased (b) Decreased
    (c) Constant (d) Zero
21. In capacitive circuit, the formula for $X_C$ is:
    (a) $X_C = 2\pi fC$ (b) $X_C = 1/2\pi fC$
    (c) $X_C = 2\pi fL$ (d) $X_C = I/V$
22. The unit of XC is:
    (a) Ohm (b) Henry
    (c) Henry/m (d) Ohm/m
23. In RL series circuit, current & voltage are:
    (a) In phase (b) Current 90º lags
    (c) Voltage 90º lags (d) Current lags between 0 & 90º
24. In RL series circuit, the impedance, Z =
    (a) V/I (b) $\sqrt{R^2 + X_L^2}$
    (c) $R^2 + X_L^2$ (d) Both a & b
25. In RL series circuit, formula to determine the current is:
    (a) $\frac{V}{R^2}$ (b) $\frac{V}{X_L^2}$ (c) $\frac{V}{Z}$ (d) $\frac{V}{R^2 + X_L^2}$
26. In RL series circuit, power factor is:
    (a) Zero (b) Less than unity
    (c) Unity (d) More than 1
27. In RL series circuit, formula to determine the P.F. is:
    (a) P/VI (b) R/Z (c) Z/R (d) Both a & b
28. The unit of impedance is:
    (a) Henry (b) Ohm (c) Ohm/m (d) Henry/m
29. In impedance triangle, base indicates:
    (a) $X_L$ (b) Impedance
    (c) Resistance (d) $X_L$ - R
30. In impedance triangle, hypotonus indicates:
    (a) $X_L$ (b) Impedance
    (c) Resistance (d) $X_L$ - R
31. In RL series circuit, the actual component of voltage is:
    (a) V cos θ (b) V sin θ
    (c) V tan θ (d) VI sin θ

32. In RC series circuit, current & voltage are:
    (a) In phase (b) Current leads 90º
    (c) Current lags 90º (d) Current leads between 0º & 90º
33. In RC series circuit, the impedance, Z =
    (a) V/I (b) $\sqrt{R^2 + X_C^2}$
    (c) $R^2 + X_C^2$ (d) Both a & b
34. In RC series circuit, formula to determine the current is:
    (a) $\frac{V}{Z}$ (b) $\frac{V}{\sqrt{R^2 + X_C^2}}$
    (c) $\frac{V}{R^2 + X_C^2}$ (d) Both a & b
35. The power factor of RC circuit is:
    (a) Always zero (b) Always unity
    (c) Between 0 & 1 (d) None
36. If the $X_C$ is less than $X_C$, then total reactance will be:
    (a) Positive (b) Negative
    (c) Anyone (d) None
37. In AC circuit, the formula to find impedance, Z =
    (a) I/V (b) V/I
    (c) $X^2 - R^2$ (d) $R^2 - X^2$
38. In AC circuit, the ratio between kW & kVA is called:
    (a) Load factor (b) Form factor
    (c) Power factor (d) Peak factor
39. Power factor of AC circuit is:
    (a) Cos θ (b) Sin θ (c) Tan θ (d) None
40. In AC circuit, component I cos θ is:
    (a) Watt full (b) Watt less
    (c) Reactive (d) Both a & c
41. In AC circuit, component I sin θ is:
    (a) Watt full (b) Watt less
    (c) Reactive (d) Both b & c
42. The unit of real power is:
    (a) Volt ampere (b) Watt
    (c) VAR (d) Both a & b
43. It is used to measure the real power of the circuit:
    (a) Voltmeter (b) Wattmeter
    (c) Energy meter (d) kVAR meter
44. It is to solve the RLC parallel method:
    (a) Vector method (b) Admittance method
    (c) Phaser algebra (d) All
45. Admittance is denoted with:
    (a) Z (b) $X_L$ (c) $X_C$ (d) Y

46. Admittance is reciprocal of:
    (a) Conductance (b) Capacitance
    (c) Impedance (d) Susceptance
47. In RLC circuit, when $X_L = X_C$ then it is called:
    (a) Impedance (b) Resonance
    (c) Conductance (d) Susceptance
48. When $X_L = X_C$ in circuit, then the frequency is:
    (a) Low (b) High
    (c) Resonant (d) Zero
49. Power factor of series resonance circuit is:
    (a) Zero (b) Lagging
    (c) Leading (d) Unity
50. In RLC series resonant circuit:
    (a) $X_L = R$ (b) $X_C = R$
    (c) $X_L = X_C$ (d) $X_C = Z$
51. In RLC series resonant circuit, resonance frequency =
    (a) $\frac{1}{2\sqrt{LC}}$ (b) $\frac{1}{2\pi\sqrt{LC}}$
    (c) $\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ (d) $\frac{1}{\sqrt{2\pi LC}}$
52. In practically, the frequency of parallel resonance =
    (a) $\frac{1}{2\sqrt{LC}}$ (b) $\frac{1}{2\pi\sqrt{LC}}$
    (c) $\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{1}{LC} + \frac{R^2}{L^2}}$
53. In parallel resonance circuit, total current =
    (a) V/R (b) V/Z
    (c) VCR/L (d) VLR/C
54. With decreasing the power factor, the current of the circuit is:
    (a) Low (b) High (c) Constant (d) Zero
55. Advantage of improving the power factor:
    (a) Saving conductor material (b) Decreased kWh
    (c) Decreased copper losses (d) All

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | d | 2. | a | 3. | d | 4. | d | 5. | c |
| 6. | c | 7. | b | 8. | a | 9. | d | 10. | a |
| 11. | a | 12. | d | 13. | b | 14. | b | 15. | c |
| 16. | a | 17. | a | 18. | d | 19. | a | 20. | b |
| 21. | b | 22. | a | 23. | d | 24. | d | 25. | c |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 26. | b | 27. | d | 28. | b | 29. | c | 30. | b |
| 31. | a | 32. | d | 33. | d | 34. | d | 35. | c |
| 36. | b | 37. | b | 38. | c | 39. | a | 40. | a |
| 41. | d | 42. | b | 43. | b | 44. | d | 45. | d |
| 46. | c | 47. | b | 48. | c | 49. | d | 50. | c |
| 51. | b | 52. | c | 53. | c | 54. | b | 55. | d |

# PART-B

**Give the short answer of the following questions**

1. Define pure resistive A.C circuit. Also write the formula to find current.
2. Write the four properties of resistive AC circuit.
3. Why the power factor of a pure resistive AC circuit is unity.
4. Define pure inductance A.C circuit. Why the current is lagging with supply voltage in the circuit.
5. What is inductance of an inductive circuit?
6. What is inductive reactance?
7. Write the four properties of pure inductive AC circuit.
8. Write four characteristics of pure capacitive A.C circuit.
9. Write four characteristics of RL A.C circuit.
10. Write four properties of R-C a c circuit.
11. Define RLC A.C series circuit. Draw its circuit diagram.
12. What do you mean by lead & lag?
13. Define impedance.
14. What is power factor?
15. Write the formulae to find XL and I in a pure inductive circuit.
16. If frequency doubled of pure inductive circuit what will effect on current of the circuit.
17. A coil has inductance 0.2H and frequency 50Hz. What is $X_L$?
18. A pure inductive circuit has $X_L$ = 62.8 ohm and V = 100 volt, what is current.
19. Write formulae to find $X_C$ and I in pure capacitive circuit.
20. A pure capacitive circuit have $X_C$ = 318.3 ohm and V = 240 volt, what is current.
21. Write the formulae of impedance and current in RL A.C circuit.
22. An inductive circuit has R = 40 ohm and $X_C$ = 62.8 ohm. What is impedance?
23. An inductive circuit has Z = 74.45 ohm and V = 500-volt, current will be.
24. Capacitive circuit has R = 4 ohm and $X_C$ = 3 ohm. Impedance will be.
25. Capacitive circuit has Z = 5 ohm and V = 50 volt. The current will be.
26. Write the formulae to calculate impedance in RLC series circuit.

27. Define phase angle, write its formulae.
28. Why the power factor of a pure resistive AC circuit is zero.
29. Define active and reactive component of current. Draw their diagram.
30. Define real or actual or active power.
31. Define apparent power.
32. An inductive circuit has R = 40 ohm and Z = 74.45 ohm. Find its power factor.
33. An inductive circuit has power 0.537, what is its phase angle.
34. Define RLC parallel circuit.
35. Enlist the methods to solve the RLC parallel AC circuit.
36. Define series resonance circuit.
37. Define resonant frequency. Write the formula to find it.
38. Write four properties of series resonance circuit.
39. What will be the current of RLC series circuit in resonance, if its R = 10 ohm and V = 200 volt.
40. Write four characteristics of parallel resonance circuit.
41. Write formulae of current and frequency of an ideal parallel resonance circuit.
42. Write formula of frequency of practical parallel resonance circuit.
43. What is reactive power?
44. What is impedance triangle, draw its diagram.
45. Write four advantages of power factor improvement.
46. Write disadvantages of low power factor.
47. Define harmonics.
48. Define true power factor. Write its formula.

## PART-C

**Give the detailed answer of the following questions.**

1. Explain the characteristics of pure resistive circuit with waveform, vector diagram.
2. Explain characteristics of a pure inductive circuit with waveform, vector diagram.
3. Explain characteristics of a pure capacitive circuit with waveform, vector diagram.
4. Explain characteristics of RL series circuit with waveform and vector diagram.
5. Explain characteristics of RC series circuit with waveform and vector diagram.
6. Explain characteristics of RLC A.C series circuit with circuit and vector diagrams.

7. Explain characteristics of RLC A.C parallel circuit with circuit diagram.

9. Define series resonance circuit and state its characteristics.

10. Define parallel resonance circuit and state its characteristics.

11. What is meant by harmonics? Explain with diagrams.

## PART-D

**SOLVE THE PROBLEMS**

**1. Calculate the reactance of a coil of inductance 0.32 H when it is connected to a 50 Hz supply. A coil has reactance of 124 ohm in a circuit with a supply of frequency 5 KHz. Determine the inductance of the coil. (100.5 ohm, 3.95 mH)**

**2. A coil has an inductance of 40mH and negligible resistance. Calculate its inductive reactance and the resulting current if connected to (a) a 240 v, 50 Hz supply and (b) a 100 v, 1 KHz supply.**
**(12.57 Ohm, 19.09 A, 0.398A)**

**3. Determine the capacitive reactance of a capacitor of 10 micro farad, when connected to a circuit of frequency; (a) 50 Hz (b) 20 KHz. (318.3-ohm, 0.796 ohm)**

**4. A capacitor has a reactance of 40 ohm when operated on a 50 Hz supply. Determine the value of its capacitance. (79.58 mF)**

**5. Calculate the current taken by a 23 micro farad capacitor when connected to a 240 v, 50 Hz supply. (1.73 A)**

**6. In a series R.L circuit the potential difference across the resistance R is 12 v and the potential difference across the inductance L is 5 v. Find the supply voltage and the phase angle current and voltage.**
**(13 V, 22.62° Lagging)**

**7. A coil has a resistance of 4 ohm and an inductance of 9.55 mH. Calculate (a) the reactance, (b) the impedance, (c) the current taken from a 240, 50 Hz supply. Determine also the phase angle between the supply voltage and current.**
**(3ohm, 5ohm, 48 A, 36.87° lagging)**

**8. A coil takes a current of 2A from a 12 V, DC supply. When connected to a 240 v, 50 Hz supply the current is 20A. Calculate the resistance, impedance, inductive reactance and inductance of the coil. (6-ohm, 12-ohm, 10.39 ohm, 33.1 mH)**

**9.** **A coil inductance 318.3 mH and negligible resistance is connected in series with a 200-ohm resistor to a 240 v, 50 Hz supply. Calculate (a) the inductive reactance of the coil, (b) the impedance of the circuit, (c) the current in the circuit, (d) the potential difference across each component and (e) the circuit phase angle.**
**(100-ohm, 223.6-ohm, 1.073 A, 107.3 A, 214.6 v, 26.57° lagging)**

**10.** **A coil consists of a resistance of 100 ohm and an inductance of 200 mH. If an alternating voltage, v given by v=200sin500t volts is applied across the coil, calculate (a) the circuit impedance (b) the current flowing (c) the potential difference across the resistance, (d) the potential difference across the inductance and (e) the phase angle between voltage and current.**
**(100-ohm, 141.4-ohm, 1 A, 100 V, 100V, 45°)**

**11.** **A pure inductance of 1.273 mH is connected in series with a pure resistance of 30 ohm. If the frequency of the sinusoidal supply is 5 KHz and the potential difference across the 30-ohm resistor is 6 V, determine the value of the supply voltage and the voltage across the 1.273 mH inductance. Draw the Phasor diagram. (10 V, 8 V)**

**12.** **A resistor of 25 ohm is connected in series with a capacitor of 45 mF. Calculate (a) the impedance and (b) the current taken from a 240 V, 50 Hz supply. Find the phase angle between the supply voltage and the current. (75.03-ohm, 3.20 A, 70.54°)**

**13.** **A capacitor C is connected in series with a 40-ohm resistor across a supply of frequency 60 Hz. A current of 3A flows and the circuit impedance is 50 ohms. Calculate (a) the value of capacitance (b) the supply voltage (c) the phase angle between the supply voltage (d) the potential difference across the resistor and (e) the potential difference across the capacitor. Draw the Phasor diagram.**
**(88.42 mF, 150 v, 36.87° leading, 120 V, 90 V)**

**14.** **A coil of resistance 5 ohm and inductance 120 mH in series with a 100-mF capacitor is connected to a 300 v, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference (c) the voltage across the coil and (d) the voltage across the capacitor.**
**(37.70-ohm, 31.83-ohm, 38.91 A, 49.58°, 1480 v, 82.45° lagging, 1239V)**

**15.** **A coil having a resistance of 10 ohm and an inductance of 125 mH is connected in series with a 60-mF capacitor across a 120-v supply. At what frequency does resonance occur? Find the current flowing at the resonant frequency. (58.12 Hz, 12 A)**

**16.** **The current resonance in a series L.C.R circuit is 100 mA. If the applied voltage is 2mV at a frequency of 200 KHz, and the circuit inductance is 50 mH, find (a) the circuit resistance (b) the circuit capacitance. (20 ohm, 0.0127 μF or 12.7 nF)**

**17.** **A coil of inductance 80 mH and negligible resistance is connected in series with a capacitor of 0.25 mF and a resistor of resistance**

**12.5 ohm across a 100 v, variable frequency supply. Determine (a) the resonant frequency and (b) the current at resonance. How many times greater than the supply voltage is the voltage across the reactance at resonance.**

**(1125.4 Hz or 1.1254 KHz, 8A, 4525.5 V)**

18. **An instantaneous current i=250sinωt mA flows through a pure resistance of 5K ohm. Find the power dissipated in the resistor.**
**(156.2 watts)**
19. **A series circuit of resistance 60 ohm and inductance 75 mH is connected to a 110 v, 60 Hz supply. Calculate the power dissipated.**
**(165 watts)**
20. **A pure inductance is connected to a 150 v, 50 Hz supply and the apparent power of the circuit is 300 VA. Find the value of the inductance. (0.239 H)**
21. **A transformer has a rated output of 200 KVA at a power factor of 0.8. Determine the rated power output and the corresponding reactive power. (160 KW, 120KVAR)**
22. **A load takes 90 KW at a power factor of 0.5 lagging. Calculate the apparent power and the reactive power. (180 KVA, 156KVAR)**
23. **A circuit consisting of a resistor in series with a capacitor takes 100 watts at a power factor of 0.5 from a 100 v, 60 Hz supply. Find (a) the current flowing, (b) the phase angle (c) the resistance (d) the impedance and (e) the capacitance. (2 A, 25-ohm, 50-ohm, 61.26 µF)**
24. **The power taken by an inductive circuit when connected to a 120 v, 50 Hz supply is 400 W and the current is 8A. Calculate (a) the resistance (b) the impedance (c) the reactance (d) the power factor and (e) the phase angle between voltage and current.**
**(6.25-ohm, 13.64-ohm, 65.37 lagging°)**
25. **A 30-mF capacitor is connected in parallel with an 80-ohm resistor across a 240 v, 50 Hz supply. Calculate (a) the current in each branch, (b) the supply current (c) the circuit phase angle (d) the Circuit impedance (e) the power dissipated and (f) the apparent power.**
**(3 A, 2.262 A, 3.757 A, 37.02 leading, 63.88-ohm, 720 W, 901.7 VA)**
26. **A capacitor C is connected in parallel with a resistor R across a 120 V, 200 Hz supply. The supply current is 2 A at a power factor of 0.6 leading. Determine the values of C and R. (1.2 A, 1.6 A, 100 ohm, 10.61 µF)**
27. **A pure inductance of 120 mH is connected in parallel with a 25 µF capacitor and the network is connected to a 100 v, 50 Hz supply. Determine (a) the branch current (b) the supply current and its phase angle, (c) the circuit impedance and (d) the power consumed.**

**(2.653 A, 0.786 A, 1.867 A, 0 W)**

**28. A 500V, 50 Hz single phase motor takes a full load current of 50A at power factor 0.8 lagging, what capacitance must be placed in parallel with the motor to raise the power factor to unity. (191μF)**

**29. Two inductive circuits of resistance 5 ohm and 8 ohm and inductances 0.02 henry and 0.01henry respectively are connected in parallel across 240 V, 50 hertz supply. Find the current taken from the supply. (55.82 A)**

**30. A circuit of resistance 12 ohms and inductive reactance 10 ohms is connected in parallel with another circuit consisting of a resistor of 20 ohms in series with a capacitor of capacitive reactance 15 ohms. Find the total current taken when this combination is connected across 220 V, 40 hertz supply. (18.25 A)**

# Chapter # 9
# POLYPHASE FUNDAMENTALS

## 9.1 TWO-PHASE GENERATION:

The elementary 2-phase 2-pole synchronous generator has a Rotor equipped with 2 coils displaced $90^o$ from each other; although shown as concentrated, they actually are distributed. When the stator is excited with dc and rotor is rotated, the sinusoidal voltages are generated in the 2 Rotor phases, displaced $90^o$ in time and having a frequency directly related to rotor speed.

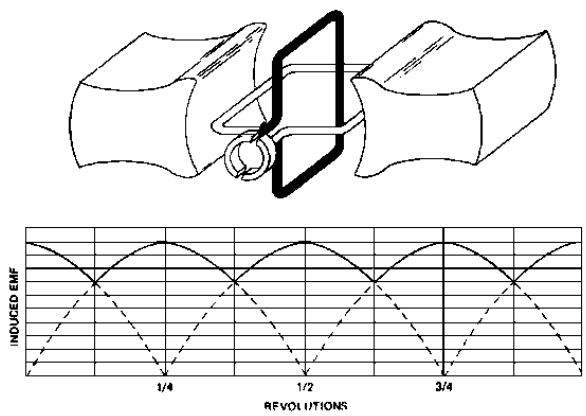
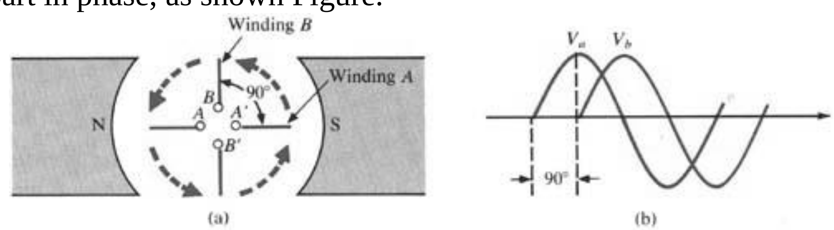
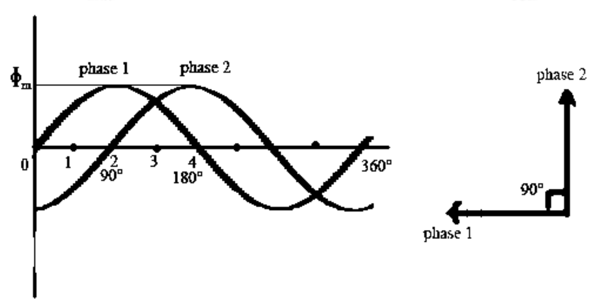



Figure below shows a basic Two-phase generator with the two loops separated by 90°. Both loops are mounted on the same rotor and therefore rotate at the same speed. Loop A is 90° ahead of loop B in the direction of rotation. As they rotate, two induced sinusoidal voltages are produced that are 90° apart in phase, as shown Figure.



## THREE-PHASE GENERATION:

The elementary 3-phase 2-pole synchronous generator has a stator equipped with 3 coils displaced 120° from each other; although shown as concentrated, they actually are distributed. When the rotor is excited with dc and rotated, the resultant field will also rotate so that sinusoidal voltages are generated in the 3 stator phases, displaced 120° in time and having a frequency directly related to rotor speed.

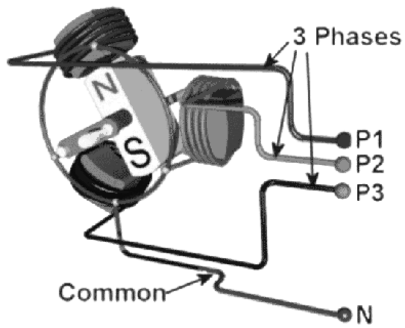
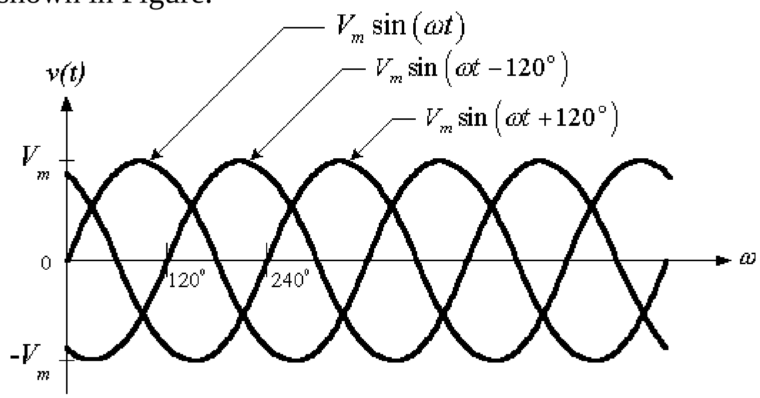



Figure shows a poly-phase generator with three separate conductor loops placed at 120° intervals around the rotor. This configuration generates three sinusoidal voltages that are separated from each other by phase angles of 120°, as shown in Figure.




## 9.2 ADVANTAGES OF POLY PHASE SYSTEM:

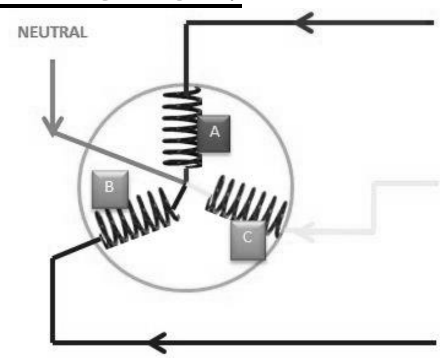
The advantages of polyphase system over single-phase systems are given below:

1. Power delivered is constant. In single phase circuit the power delivered is pulsating and objectionable for many applications.
2. For a given frame size a polyphase machine gives a higher output than a single-phase machine.
3. Polyphase induction motors are self-starting and are more efficient. Single phase motor has no starting torque and requires an auxiliary means for starting.
4. Comparing with single phase motor, three phase induction motor has higher power factor and efficiency. Three phase motors are very robust,

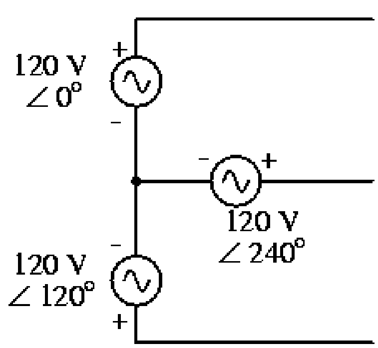
relatively cheap, generally smaller, have self-starting properties, provide a steadier output and require little maintenance compared with single phase motors.

5. For transmitting the same amount of power at the same voltage, a three-phase transmission line requires less conductor material than a single-phase line. The three-phase transmission system is so cheaper. For a given amount of power transmitted through a system, the three-phase system requires conductors with a smaller cross-sectional area. This means a saving of copper and thus the original installation costs are less.
6. Polyphase motors have uniform torque whereas most of the single-phase motors have pulsating torque.
7. Parallel operation of three-phase generators is simpler than that of single-phase generator.
8. Polyphase system can set up rotating magnetic field in stationary windings.

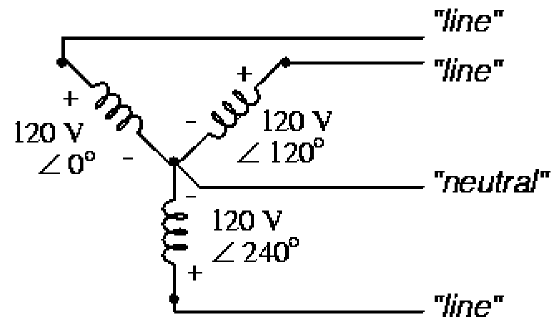
## 9.3 STAR CONNECTION:



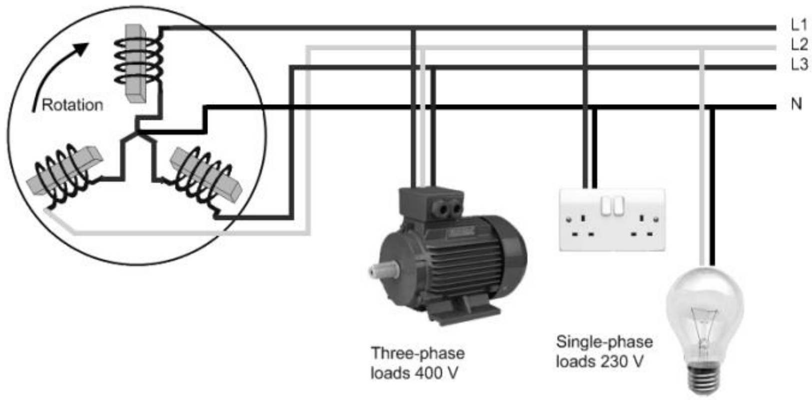

The configuration of voltage sources is characterized by a common connection point joining one side of each source is commonly known as the "Y" (or "star") configuration. (Figure below)



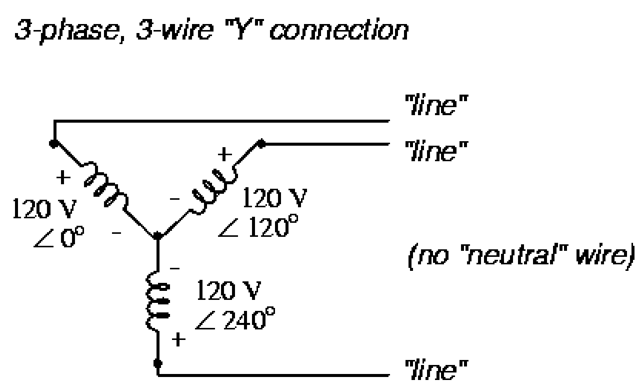

If we draw a circuit showing each voltage source to be a coil of wire (alternator or transformer winding) and do some slight rearranging, the "Y" configuration becomes more obvious in Figure below.




Three-phase, four-wire “Y” connection uses a "common" fourth wire.



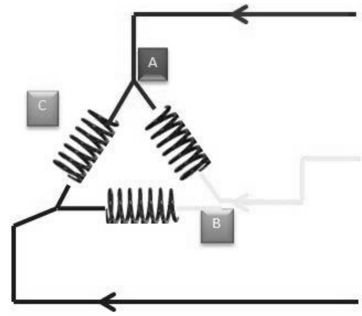

The three conductors leading away from the voltage sources (windings) toward a load are typically called lines, while the windings themselves are typically called phases. In a Y-connected system, there may or may not (Figure below) be a neutral wire attached at the junction point in the middle, although it certainly helps alleviate potential problems should one element of a three-phase load fail open, as discussed earlier.



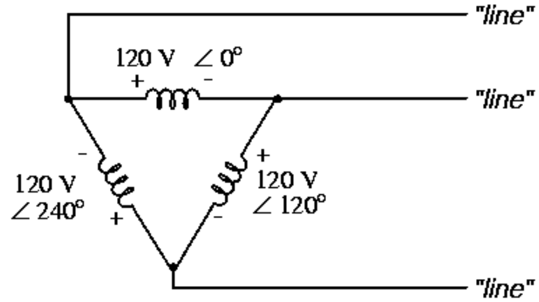

Three-phase, three-wire “Y” connection does not use the neutral wire.

## DELTA CONNECTION:

Another configuration is known as the "Delta," for its geometric resemblance to the Greek letter of the same name (Δ).




Take close notice of the polarity for each winding in Figure below.




Three-phase, three-wire Δ connection has no common.

## 9.4 RELATIONSHIP BETWEEN PHASE AND LINE CURRENT IN STAR CONNECTION:

In star connection, line current equal to phase currents (As line and phase is being in series). If, $I_L$ represents value of line current and $I_{ph}$ represents the value of phase current then,

$I_L = I_{ph}$

## RELATION BETWEEN PHASE & LINE VOLTAGES IN STAR CONNECTION:

Y-connected sources and loads always have line voltages greater than phase voltages. The potential difference between lines 1 & 2 is

$V_{RY} = E_R - E_Y$ (Vector difference)

Hence, $V_{RY}$ is found by compounding $E_R$ and $E_Y$ reversed and its value is given by the diagonal of the parallelogram of figure as shown below. The angle between $E_R$ and $E_Y$ reversed (i.e. $- E_Y$) is 60º.

Hence if $E_R = E_Y = E_B = E_{ph}$

$V_{RY} = 2 E_{ph} \cos (60^\circ / 2)$

$= 2 E_{ph} \cos 30^\circ$

$= 2\ E_{ph}\frac{\sqrt{3}}{2}$

$= \sqrt{3}\ E_{ph}$

Similarly, $V_{YB} = E_Y - E_B$ (Vector difference)

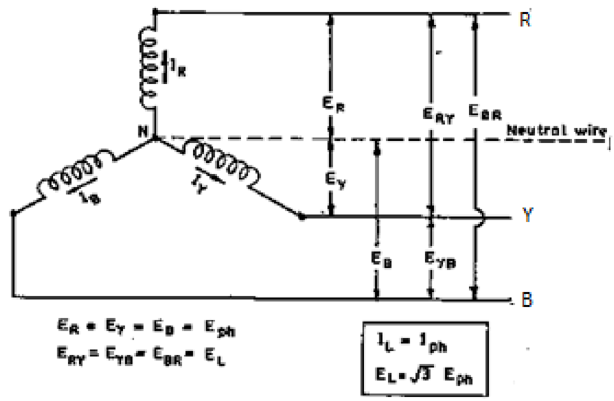
$= \sqrt{3}\ E_{ph}$

And $V_{BR} = E_B - E_R$

$= \sqrt{3}\ E_{ph}$

Now $V_{RY} = V_{YB} = V_{BR}$ = Line Voltage, say, $V_L$

Hence, in star connection $V_L = \sqrt{3}\ E_{ph}$




## RELATION BETWEEN LINE AND PHASE VOLTAGE IN DELTA CONNECTION:

Because each pair of line conductors is connected directly across a single winding in a Δ circuit, the line voltage will be equal to the phase voltage. If $V_L$ represents value of line Voltage and $V_{ph}$ represents the value of phase Voltages then,

$V_L = V_{ph}$

## RELATION BETWEEN PHASE & LINE CURRENTS:

Because each line conductor attaches at a node between two windings, the line current will be the vector difference of the two joining phase currents.

The Line current of line 1 is

$I_1 = I_R - I_B$ (Vector difference)

Hence, $I_1$ is found by compounding $I_R$ and $I_B$ reversed and its value is given by the diagonal of the parallelogram of figure as shown below. The angle between $I_R$ and $I_B$ reversed (i.e. $-I_B$) is 60º. Hence if $I_R = I_Y = I_B = I_{ph}$

$I_1 = 2\ I_{ph} \cos(60^\circ / 2)$

$= 2\ I_{ph} \cos 30^\circ$

$= 2\ I_{ph}\frac{\sqrt{3}}{2}$

$= \sqrt{3}\ I_{ph}$

Similarly, $I_2 = I_Y - I_R$ (Vector difference)

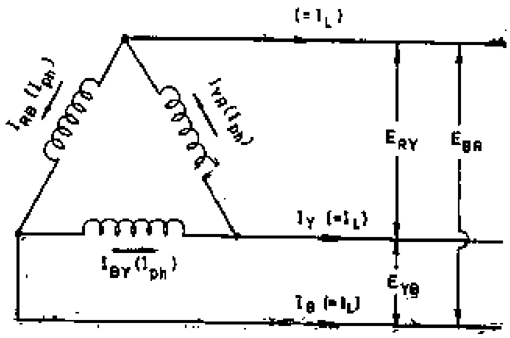
$= \sqrt{3}\, I_{ph}$

And $I_3 = I_B - I_Y$

$= \sqrt{3} I_{ph}$

Now $I_1 = I_2 = I_3$ = Line Current, say, $I_L$

Hence, in delta connection $I_L = \sqrt{3} I_{ph}$




## 9.5 COMPARISON OF STAR & DELTA CONNECTION:

With a delta connection, picture a triangle. Each side of the triangle is a transformer or motor winding. Call the 3 corners of the triangle A, B, and C. There are only 3 wires except for a safety ground, which has no connection to, nor is any part of, the power transmission service lines. All 3 windings have the same voltage, say for example 208V. If you take a meter, you will measure 208V between A and B, between B and C, and between C and A.

With a wye connection (also called a star connection) takes our triangle above and uses your wire cutters (literally) to cut the triangle apart at the corners. Rearrange the three sides (transformer or motor windings) to form a Y. Where the three sides join in the center, connect a fourth wire. Call this wire "neutral". Call the three ends A, B, and C. Connect the 3 supply service lines to A, B, and C.

Now if you measure with your meter you will see 120V between the neutral and any single phase (A, B, or C), but you will still measure 208V between A and B, B and C, and C and A. The 208V is used to power three-phase loads and the 120V is very handy if you need to power a 120V single-phase load, such as a toaster! In a wye system, the supply service line-to-line voltage is 1.73 times the voltage from any phase to neutral. Delta power is used for motors, 3-phase heaters; anywhere you don't need a neutral. The biggest use of delta is in power transmission. Way too expensive to run a fourth wire all those miles, especially since a 3-wire delta transmits the same amount of power. Look up at a transmission tower and you see three phases and a ground wire for lightning suppression. No neutral. At the destination, (a distribution transformer outside the home or business), the primary of the transformer is wired delta and the secondary is wired wye. This creates the neutral that can be used to derive single-phase power where needed. The neutral is grounded at the service entrance. Wye systems must be used where you have single phase loads that you must feed. Interestingly, if you hide the neutral, you really can't tell the difference between star and delta systems. The phase-to-phase voltage is exactly the same. Most of the time 3-phase motors (which are internally delta-connected and have no neutral) are fed from a wye-connected source, simply omitting the neutral.

The three phase windings in a generator / transformer /motor can be interconnected in two ways. If the similar ends of the coils are connected together and the other ends are connected to the 3 incoming supply service lines that is called "star connection". The internally-connected point is brought out as the neutral connection. The service line "line-to-line" voltage (between any 2 lines) is 1.732 times the voltage between any line and neutral. Thus, we can get two voltages for distribution within a building or site to supply lighting and power. However, the service line currents are the same as the phase winding currents. If the dissimilar ends of each winding are labeled A and B and the three windings are then connected together B->A, B->A, B->A, and the 3 incoming supply service lines are then connected to the junctions, it is called DELTA. Here the values of line voltage and phase voltage are the same while the service line currents are 1.732 times the phase winding currents.

## 9.6 POWER EQUATION OF THREE PHASE:

We will first examine three-phase power in the context of a wye-load; then we'll examine a delta load.

## STAR / WYE CONNECTED LOAD:

Suppose that each phase has impedance .

$\mathbf{Z}_{ph} = Z\angle\theta_{ph} = R_{ph} + j\,X_{ph}$

Then the active (real)

Power per phase ($P_{ph}$) is given

$$P_{ph} = V_{ph} I_{ph} \cos\theta$$
$$= I_{ph}^2 R_{ph}$$
$$= \frac{V_{R^2}}{R_{ph}} \text{ (Phase Power)}$$

Because we are considering a balanced system, the power per phase ($P_{ph}$) is identical in all three phases, and thus the total active power ($P_T$) is simply,

$$P_T = 3P_{Ph}$$
$$P_T = P_{an,} + P_{bn}, + P_{cn,} = 3P_{ph}$$

Using line voltage ($V_L = \sqrt{3}V_{ph}$) and line current ($I_L = I_{ph}$), we have

$$P_T = 3P_{Ph}$$
$$= 3V_{ph}\, I_{ph} \cos\theta$$

$= 3 \text{ x } \frac{V_L}{\sqrt{3}} I_L \text{Cos } \theta$

$= \sqrt{3} \text{ x } \sqrt{3} \text{ x } \frac{V_L}{\sqrt{3}} I_L \text{Cos } \theta$

$= \sqrt{3} V_L I_L \text{Cos } \theta$

## MESH / DELTA CONNECTED LOAD:

Suppose that each phase has impedance .

$\mathbf{Z}_{ph} = Z\angle\theta_{ph} = R_{ph} + j\,X_{ph}$

Then the active (real)

Power per phase ($P_{ph}$) is given

$P_{ph} = V_{ph} I_{ph} \text{Cos } \theta$

$= I_{ph}^2 R_{ph}$

$= \frac{V_{R^2}}{R_{ph}}$ (Phase Power)

Because we are considering a balanced system, the power per phase ($P_{ph}$) is identical in all three phases, and thus the total active power ($P_T$) is simply,

$P_T = 3P_{Ph}$

$P_T = P_{an,} + P_{bn}, + P_{cn,} = 3P_{ph}$

Using line current ($I_L = \sqrt{3} I_{ph}$) and line voltage ($V_L = V_{ph}$), we have

$P_T = 3P_{Ph}$

$= 3V_{ph} I_{ph} \text{Cos } \theta$

$= 3 \text{ x } V_L \text{X } \frac{I_L}{\sqrt{3}} \text{Cos } \theta$

$= \sqrt{3} \times \sqrt{3} \times \frac{I_L}{\sqrt{3}} V_L \cos \theta$

$= \sqrt{3} V_L I_L \text{Cos } \theta$

Same as in case of star connected load, hence in 3 phase star or delta connected load;

$P_T = \sqrt{3} V_L I_L \text{Cos } \theta$

This is three phase power equation**.** The above expression of power shows that the total instantaneous power is constant and equal to three times of the real power per phase. In three phase line values of voltage and current are often used hence, in this case;

$w_{applied} = \sqrt{3} v_l i_l \cos \theta$

where

$w_{applied}$ = real power (w, watts)

$v_l$= line voltage (v, volts)

$i_l$ = line current (a, amps)

cos θ = power factor (usually value 0.7 - 0.95)

For purely resistive load: Cos θ = 1

Resistive loads convert current into other forms of energy, such as heat
Inductive loads use magnetic fields like motors, solenoids, and relays

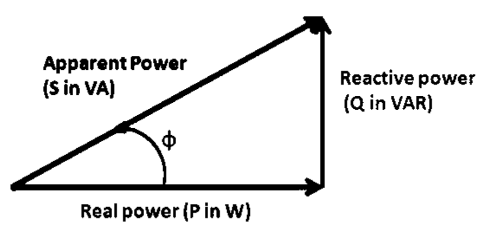
In case of single phase power expression we found that there are both reactive power and active power components, but in case of three phase power expression, the instantaneous power is constant. Actually in three phase system, the reactive power in each individual phase is not zero but sum of them at any instant is zero. Reactive power is the form of magnetic energy, flowing per unit time in an electric circuit. Its unit is VAR (Volt Ampere Reactive). This power can never be used in an AC circuit. However, in an electrical DC circuit it can be converted into heat as when a charged capacitor or inductor is connected across a resistor, the energy stored in the element get converted to heat. Our power system operates on AC system and most of the loads used in our daily life, are inductive or capacitive, therefore reactive power is a very important concept from electrical perspective.

The electrical power factor of any equipment determines the amount of reactive power it requires. It is the ratio of real or true power to the total apparent power required by an electrical appliance. These powers can be calculated as,

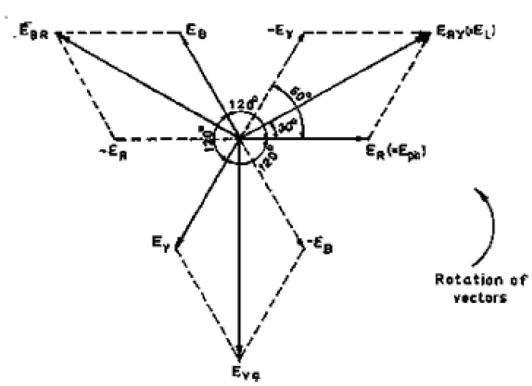
Apparent Power = $S = \sqrt{3}V_L I_L$ (VA or KVA)

Real Power = $P = \sqrt{3}V_L I_L \cos\Phi$ (WATT OR KW)

REACTIVE POWER = $Q = \sqrt{3}V_L I_L \sin\Phi$ (VAR OR KVAR)

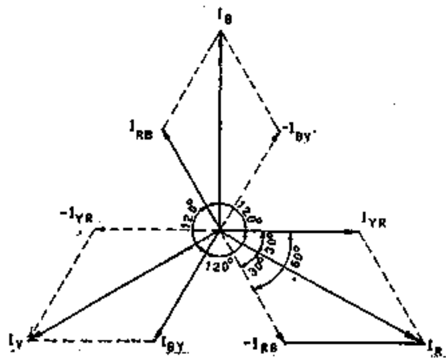



## 9.7: VECTOR DIAGRAM OF STAR CONNECTION




. Vector diagram for star connected net-work.

## VECTOR DIAGRAM OF DELTA CONNECTION:




## 9.9 EXAMPLE - PURE RESISTIVE LOAD

For purely resistive load and **POWER FACTOR = 1**, the real power in a **415 VOLTAGE 20 AMPS** circuit can be calculated as

$W_{applied} = \sqrt{3} V_L I_L \mathbf{Cos\ \Phi} = 1.732 \times 415 \times 20 \times 1$
$= \underline{14400}$ W
$= \underline{14.4}$ kW

**EXAMPLE:**

**A 6600 V, three phase star connected generator supplies 4000KW at a power factor of 0.8 lagging. Calculate the line and phase currents?**

**Solution:**

$V_L = 6600V$, $P_T = 4000$ KW = 4000000-Watt, $\cos\theta = 0.8$ lagging

$I_L = ?A$ and $I_{PH} = ?A$

$$P_T = \sqrt{3}\, V_L\, I_L \cos\theta$$

$$I_L = \frac{P_T}{\sqrt{3} V_L \cos\theta} = \frac{4000000}{\sqrt{3} \times 6600 \times 0.8} = 437.5\ A$$

$$I_{PH} = I_L = 437.5A$$

**EXAMPLE:**

**A 6600 V, three phase delta connected motor receiving 4000KW at a power factor of 0.8 lagging. Calculate the line and phase currents?**

**Solution:**

$V_L = 6600V$, $P_T = 4000$ KW = 4000000-Watt, $\cos\theta = 0.8$ lagging

$I_L = ?A$ and $I_{PH} = ?A$

$$P_T = \sqrt{3}\, V_L\, I_L \cos\theta$$

$$I_L = \frac{P_T}{\sqrt{3} V_L \cos\theta}$$

$$= \frac{4000000}{\sqrt{3} \times 6600 \times 0.8} = 437.5\ A$$

$$I_{PH} = \frac{I_L}{\sqrt{3}} = \frac{437.5}{\sqrt{3}} = 252.598A$$

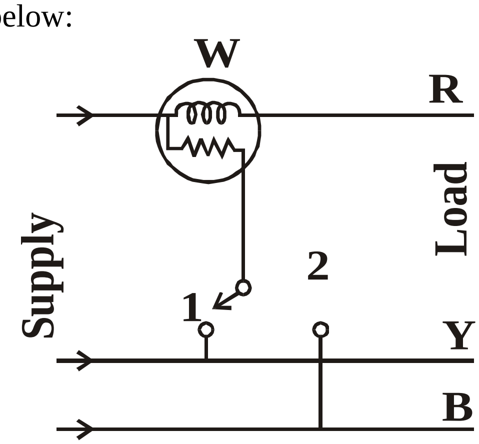
## 9.10 MEASUREMENT OF THREE PHASE POWER:

Measurement of three phase power in three phase circuits on the basis of number of wattmeter's used; we have three methods;

1. Three wattmeter method
2. Two wattmeter method
3. Single wattmeter method

### MEASUREMENT OF THREE PHASE POWER BY ONE WATTMETER METHOD:

Limitation of this method is that it cannot be applied on unbalanced load. So, under this condition we have $I_1=I_2=I_3=I$ and $V_1=V_2=V_3=V$
Diagram is shown below:




Two switches are given which are marked as 1-3 and 1-2, by closing the switch 1-3 we get reading of wattmeter as

$$P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} \times VI \cos(30 - \phi)$$

Similarly, the reading of wattmeter when switch 1-2 is closed is

$$P_2 = V_{12} I_1 \cos(30 + \phi) = \sqrt{3} \times VI \cos(30 + \phi)$$

$$Total\ power\ is\ P_1 + P_2 = 3VI\cos\phi$$

## 9.11 MEASUREMENT OF THREE PHASE POWER BY TWO WATTMETERS METHOD:

In this method we have two types of connections

1- Star connection of loads.
2- Delta connection of loads.

When the star connected load, the diagram is shown in below-

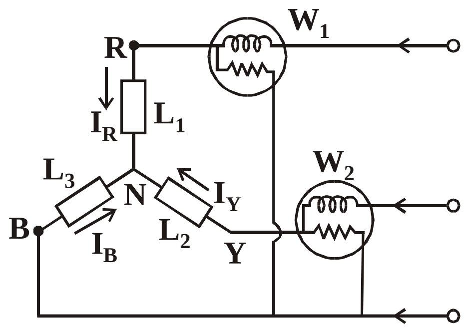



Fig. (a)

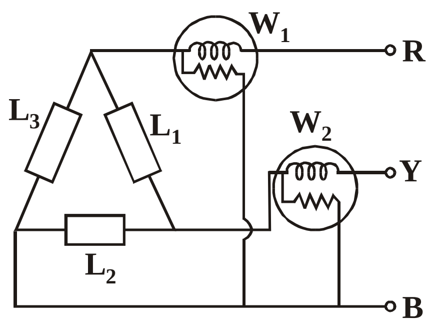



Fig. (b)

For star connected load clearly the reading of wattmeter one is product phase current and voltage difference ($V_1$-$V_3$). Similarly, the reading of wattmeter two is the product of phase current and the voltage difference ($V_2$-$V_3$). Thus, the total power of the circuit is sum of the reading of both the wattmeter. Mathematically we can write

$$
\begin{aligned}
P_T &= P_1 + P_2 = I_1 (V_1 - V_3) + I_2 (V_2 - V_3) \\
&= I_1V_1 - I_1 V_3 + I_2 V_2 - I_2 V_3 \\
&= I_1V_1 + I_2 V_2 - I_1 V_3 - I_2 V_3 \\
&= I_1V_1 + I_2 V_2 - V_3 (I_1 + I_2)
\end{aligned}
$$

But we have $I_1+I_2+I_3=0$, hence putting the value of $I_1+I_2=-I_3$

We get total power as $V_1I_1+V_2I_2+V_3I_3$

When delta connected load, the diagram is shown in below

The reading of wattmeter one can be written as

$$P_1 = -V_3( I_1 - I_3 )$$

The reading of wattmeter two is

$$P_2 = -V_2( I_2 - I_1 )$$

$$Total\ power\ is\ P = P_1 + P_2 = V_2I_2 + V_3I_3 - I_1( V_2 + V_3 )$$

But $V_1+V_2+V_3=0$, hence expression for total power will reduce to $V_1I_1+V_2I_2+V_3I_3$

When using the two-watt meter method, it is important to note that the reading of one wattmeter should be reversed if the power factor of the system is less than 0.5. In such a case, the leads of one wattmeter may have to be reversed in order to get a positive reading. In the case of a power factor less than 0.5, the readings must be subtracted instead of being added.

The power factor of the three-phase system, using the two-wattmeter method (W1 and W2) can be calculated as follows:

$$\tan ø = \sqrt{3}\frac{(W_{1-} W_2 )}{(W_{1+} W_2 )}$$

## 9.12 MEASUREMENT OF THREE PHASE POWER BY THREE WATTMETER METHOD:

The circuit diagram is shown below-

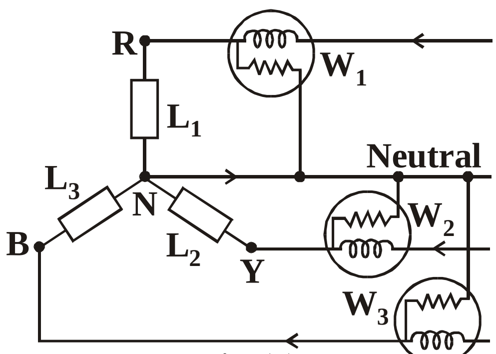


Fig. (a)

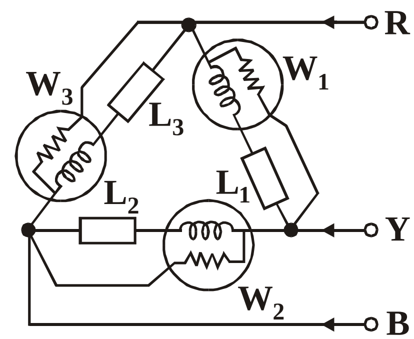


Fig. (b)

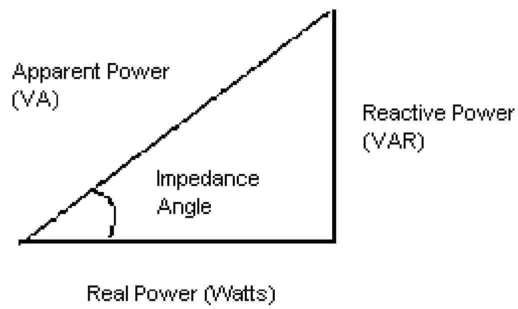
Here, it is applied to three phase four wire systems, current coil of all the three-wattmeter marked as one, two and three are connected to respective phases marked as one, two and three. Pressure coils of all the three-wattmeter are connected to common point at neutral line. Clearly each wattmeter will give reading as product of phase electric current and line voltage which is phase power. The resultant sum of all the readings of wattmeter will give the total power of the circuit. Mathematically we can

$$P = P_1 + P_2 + P_3 = V_1I_1 + V_2I_2 + V_3I_3$$

## 9.13 MEASUREMENT OF REACTIVE POWER:

A power measurement application requires one measurement device to capture the voltage across the terminals of a load, and the second device to capture the current going through the load. However, the actual power calculation depends on the resistive and reactive components (capacitors and/or inductors) in the circuit. The power dissipation in a purely resistive circuit is always a function of the voltage drop and current draw through the circuit.

Reactive circuits appear to function like resistive circuits because they produce voltage drops and draw current. However, reactive circuits actually store or return power. The reactive components cause a phase shift (up to 90 degrees) between the voltage and current waveforms, which reduce the overlap between the two curves, and effectively deliver less power to the loads. This phenomenon is represented by three different power measurements: reactive power, apparent power, and real power. These three power measurements have a phase relationship that can be visualized in the power triangle, shown below.




Reactive power is the measurement of a circuit's reactance (X). A purely reactive circuit dissipates zero real power because the power is absorbed by the circuit and returned to the AC source. This circuit results in 90 degrees phase shift between the voltage and current waveforms, as shown in Figure below. A circuit with resistive and reactive components will both dissipate power and return power to the AC source. The current and voltage waveforms will have a phase shift between 0 and 90 degrees.

Reactive power is represented with Q and has a unit measure of Volt-Amps-Reactive (VAR). You use the following formula to calculate reactive power:

Reactive Power (Q) = SQRT[( Apparent power)2 - (Real Power)2] VAR

To accurately measure reactive power, you need the following capabilities:

- Voltage and current waveform acquisition capability
- Simultaneous acquisition of both measurement waveforms
- Both measurement devices must acquire simultaneously
- Analysis functions

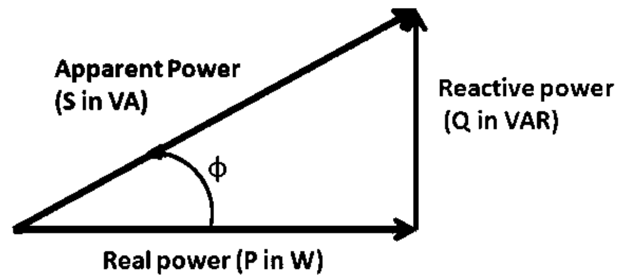
Traditionally, this measurement functionality was only found in specialized power meters because a traditional digital multi meter has asynchronous clocks and doesn't provide waveform acquisition capabilities. However, using the modern digital multi meter architecture, which boasts digitizer capability, along with analysis functions, we can build a true power measurement system.

## 9.14 SOLUTION OF POWER FACTOR PROBLEMS:

## EXAMPLE –1

A three-phase induction motor is consuming 6.5 KW active power and 4.5 KVAR Reactive power. Calculate power factor of motor.

Active / Real Power = P = 6.5 KW
Reactive Power = Q = 4.5 KVAR
Power Factor = Cos ø = ?

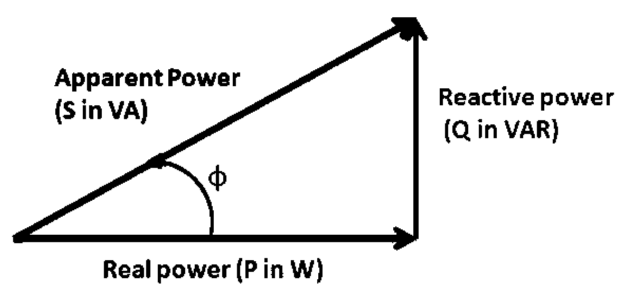



Considering the above power triangle,
Tan ø = Q / P = 4.5 / 6.5 = 0.6923
Ø = $\tan^{-1}$0.6923 = 34.695°
Cos ø = Cos 34.695°= 0.8222
Power Factor of load = 0.8222 Lagging

## EXAMPLE –2

A three-phase synchronous motor is consuming connected across 400 Volt, taking current of 12 Amperes. A three-phase wattmeter showing 5.5 KW reading. Calculate power factor of motor.

Active / Real Power = P = 5.5 KW
$V_L$ = 400 Volt
$I_L$ = 12 Amperes
Power Factor = Cos ø = ?
Apparent Power = S = $\sqrt{3}V_LI_L$
= 1.732 x 400 x 12
= 8313.6 VA
= 8313.6 / 1000 = 8.313 KVA




Considering the above power triangle,
$Q^2$ = $S^2 – P^2$
= $8.313^2 – 5.5^2$
= 69.116 – 30.25
= 38.866
Q = $\sqrt{38.866}$
= 6.234 KVAR
Tan ø = Q / P = 6.234 / 5.5 = 1.134
Ø = $\tan^{-1}$0.6923 = 48.581°
Cos ø = Cos 48.581°= 0.662
Power Factor of load = 0.662 Leading

## 9.15 PHASE SEQUENCE:

The sequence in which three phase voltages attain their positive maximum values is defined as the phase sequence. It refers to the relation between the voltages or currents in three phase system. Consider the three phases as red-R, yellow-Y and blue-B phases.

## PHASE SEQUENCE METER:

Phase sequence indicators / meters are the indicator that determines the phase sequence of the three-phase supply system. When conventional three phase supply (i.e. RYB) is given to the induction motor, suppose the direction of the rotation of the rotor is in clockwise direction. Now if the phase sequence is reversed, the rotor will rotate in the anticlockwise direction. Thus, the direction of rotation of rotor depends on the phase sequence.

The types of phase sequence indicators are:

a. Rotating type.

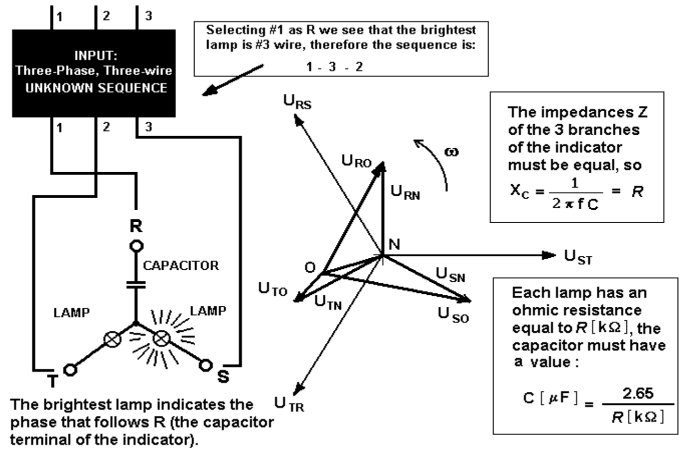
b. Static type.

### (a) ROTATING TYPE PHASE SEQUENCE INDICATORS:

It works on the principle of induction motors. In this coil are connected in star form and the supply is given from three terminal marked as RYB as shown in the figure. When supply is given the coils produces the rotating magnetic field and these rotating magnetic fields produces eddy emf in the movable aluminium disc as shown in the diagram. These eddy emf produces eddy current on the aluminium disc, eddy currents interact with the rotating magnetic field due this a torque is produced which causes the light aluminium disc to move. If the disc moves in the clockwise direction then chosen sequence is RYB and if the direction of rotation is in anticlockwise the sequence is reversed.

## (b) STATIC TYPE PHASE SEQUENCE INDICATORS:

Given below is the arrangement of static type indicator:




If the phase sequence is RYB, then the lamp B will glow brighter than the lamp A, and – if phase sequence is reversed – then the lamp A will glow brighter than the lamp B, the arrangement is shown in circuit diagram.

## 9.16 ADVANTAGES OF THREE-PHASE OVER SINGLE-PHASE SUPPLY:

A three-phase supply have following advantages over single-phase supply;

1. A three-phase supply provides greater power density than a one-phase supply at the same amperage, keeping wiring size and costs lower.
2. A three-phase supply makes it easier to balance loads.
3. A three-phase supply minimize harmonic currents.
4. A three-phase supply minimize the need for large neutral wires as in neutral wire only out of balance current flow.

## 9.17 THREE PHASE BALANCED LOAD PROBLEM SOLUTION:

## EXAMPLE 1:

**Three coils, each having a resistance of 10Ω and an inductance of 0.02 Henry, are connected (a) In Star (b) In Delta, to a three phase, 50 Hz supply, the line voltage being 500 V. Calculate for each case the line current and the total power.**

**Solution:**

$X_{L\text{ per phase}} = 2\pi F L = 2 \times 3.142 \times 50 \times 0.02 = 6.2832\ \Omega$

$Z_{PH} = \sqrt{R^2 + XL^2} = \sqrt{10^2 + 6.2832^2} = 11.81\Omega$

$\text{Cos}\ \theta = \frac{R}{Z_{PH}} = \frac{10}{11.81} = 0.847$ Lagging

(a) Star connection

$V_{PH} = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}} = 288.7$ V

$I_{PH} = I_L = \frac{V_{PH}}{Z_{PH}} = \frac{288.7}{11.81} = 24.44$ A

$P_T = \sqrt{3}\ V_L\ I_L \cos\theta$

$= 1.732 \times 500 \times 24.44 \times 0.847 = 17930$ W $= 17.93$KW

(b) Delta connection

$V_{PH} = V_L = 500$V

$I_{PH} = \frac{V_{PH}}{Z_{PH}} = \frac{500}{11.81} = 42.34$ A

$I_L = \sqrt{3}\ I_{PH} = 1.732 \times 42.34 = 73.31$A

$P_T = \sqrt{3}\ V_L\ I_L \cos\theta$

$= 1.732 \times 500 \times 73.31 \times 0.847 = 53750$ W $= 53.75$KW

## EXAMPLE 2:

**A three-phase system supplies 25 KW at a power factor of 0.8, the line voltage being 250 Volts. Calculate the Line current and phase current when the load is star connected.**

**P = 25 KW = 25 x 1000 = 25000 W**

Power factor = $\cos\theta = 0.8$

$V_L = 250$ Volts

$I_L = ?$

$I_{Ph} = ?$

In star connection

$P = \sqrt{3}\ V_L\ I_L \cos\theta$

$25000 = 1.732 \times 250 \times I_L \times 0.8$

$= 346.4 \times I_L$

$I_L = 25000 / 346.4 = 72.17$ Ampere

$I_{Ph} = I_L = 72.17$ Ampere

## EXAMPLE 3:

**A three-phase system supplies 20 KW at a power factor of 0.9, the line voltage being 400 Volts. Calculate the Line current and phase current when the load is delta connected.**

P = 20 KW = 20 x 1000 = 20000 W

Power factor = $\cos\theta = 0.9$

$V_L = 400$ Volts

$I_L = ?$

$I_{Ph} = ?$

In Delta connection

$P = \sqrt{3}\ V_L\ I_L \cos\theta$

$20000 = 1.732 \times 400 \times I_L \times 0.9$

$= 623.52 \times I_L$

$I_L = 20000 / 623.52 = 32.06$ Ampere

$I_{Ph} = I_L / \sqrt{3} = 32.06 / \sqrt{3} = 18.52$ Ampere

# EXERCISE # 09

## PART-A

Encircle the correct answer.

1. Poly phase system consists of:
   (a) One phase (b) Two phase
   (c) Three phase (d) Both b & c
2. No of winding sets used in two phase emf generation are:
   (a) Two (b) Three (c) Four (d) Six
3. The angle between winding sets of alternator rotor in two phase emf generation is:
   (a) 45° (b) 90° (c) 120° (d) 180°
4. If the voltage across a phase and neural are 230V then the voltage across two phases of two-phase supply will be:
   (a) 460 (b) $230 \times \sqrt{5}$
   (c) $230 \times \sqrt{2}$ (d) $230 \times \sqrt{3}$
5. In poly phase EMF generation, the value of EMF and Frequency between each phase:
   (a) Different (b) Same
   (c) Opposite (d) Greater
6. In poly phase system, angular displacement among three phases:
   (a) 90º (b) 120º (c) 360º/2 (d) 0º
7. The voltage between a phase and neutral are called:
   (a) Line voltage (b) Phase voltage
   (c) Phase to phase voltage (d) Both a & b
8. The voltage between two phases (lines) are called:
   (a) Line voltage (b) Phase voltage
   (c) Phase to phase voltage (d) Both a & b
9. The current flowing in any phase is called:
   (a) Line current (b) Phase current
   (c) Phase to phase current (d) Both a & b
10. The current flowing in a winding or set of coils is called:
    (a) Line current (b) Phase current
    (c) Phase to phase current (d) Both a & b
11. Winding sets in three phase generation are:
    (a) 6 (b) 4 (c) 2 (d) 3
12. Advantages of three phase system over single-phase system:
    (a) Constant power (b) Self-start of three phase motor
    (c) Constant torque (d) All of these
13. To measure, real power this meter is used:
    (a) kW meter (b) kWH meter
    (c) kVAR meter (d) V & A meter
14. In three phase system this phase sequence is used:
    (a) R,Y,B (b) Y,B,R (c) A,B,C (d) R,B,Y

15. The device used to find the phase sequence in three phase supply is:
    (a) Tachometer (b) Phase sequence meter
    (c) Oscilloscope (d) All of these
16. The phase voltage of a four wire three phase star connected system in 110v, the line voltage is:
    a) 440V (b) 330V (c) 191V (d) 110V
17. A four wire three phase star connected system has a line current of 10A. The phase current is:
    (a) 40A (b) 10A (c) 20A (d) 30A
18. In three phase star connection:
    (a) $I_L = I_{Ph}$ (b) $I_L = I_{Ph} \times \sqrt{3}$
    (c) $I_L = \frac{I_{Ph}}{\sqrt{3}}$ (d) $I_{Ph} = \frac{I_L}{\sqrt{3}}$
19. In three phase connection phase voltage =
    (a) VL (b) $V_L/\sqrt{3}$ (c) $V_L \times \sqrt{3}$ (d) $V_L \times 3$
20. In three phase delta connection:
    (a) $I_L = I_{Ph}$ (b) $I_L = I_{Ph} \times \sqrt{3}$
    (c) $I_L = I_{Ph} / \sqrt{3}$ (d) $I_L = I_{Ph} \times \frac{I_{Ph}}{\sqrt{3}}$
21. In three phase delta connection:
    (a) $V_L = V_{Ph}$ (b) $V_L = V_{Ph} \times \sqrt{3}$
    (c) $V_L = V_{Ph} / \sqrt{3}$ (d) $V_{Ph} = V_L \times \sqrt{3}$
22. In three phase circuit to measure the reactive power, this method is used:
    (a) 3 wattmeter method (b) 2 wattmeter method
    (c) 1 wattmeter method (d) Both b & c
23. By using to watt meter in three phase circuit, the formula to find power factor is:
    (a) $\tan\theta = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$ (b) $\cos\theta = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$
    (c) $\tan\theta = \frac{\sqrt{3}(W_1 + W_2)}{W_1 - W_2}$ (d) $\cos\theta = \frac{\sqrt{3}(W_1 + W_2)}{W_1 - W_2}$

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | a | 3. | b | 4. | c | 5. | b |
| 6. | b | 7. | b | 8. | a | 9. | a | 10. | b |
| 11. | d | 12. | d | 13. | a | 14. | a | 15. | b |
| 16. | c | 17. | b | 18. | a | 19. | b | 20. | b |
| 21. | a | 22. | d | 23. | a | | | | |

# PART-B

Give the short answer of the following.

1. What are poly phase AC circuits?
2. Write four advantages of two-phase system over a single-phase system.
3. Define phase sequence.
4. What are phase indicators?
5. Explain briefly how a three-phase supply is generated.
6. State the national standard phase sequence for a three-phase supply.
7. What is difference between phase voltage & line voltage?
8. Draw the diagram of Delta Connection.
9. Compare the star connection & delta connection with 4 points.
10. Write the methods to measure the power of 3-phase load.
11. If the phase current in delta is 32 A, what will be the line current?
12. If the line current in star connection 72.16 A, what will be the phase current?
13. Two-watt meters are used to measure the power of 3-phasr circuit of the reading of meter's are 30 kW and 10 kW. What is the total power of the circuit?
14. Three loads each of resistance 3W, are connected in star to a 415 V three phase supply. Determine the system phase voltage.
15. State the relationship between line & phase currents and line & phase voltages for a delta connection system. In Delta connection.
16. State two formulas for determine the power dissipated in the load of a 3-phase balanced system.
17. By what method may power by measured in a 3-phase system.
18. State a formula from which power factor may be determined for a balanced system when using the two-wattmeter method for power measurement.
19. State the relation-ship between line & phase currents and line & phase voltages for a star connected system.

# PART-C

Give the detailed answer.

1) What is meant by poly phase, what are its advantages over single phase?
2) How three phase E.M.F is produced, explain it?
3) What is meant by star connection? Derive the relation between line and phase values in star connection.
4) What is meant by delta connection? Derive the relation between line and phase value in delta connection.
5) Derive the phaser formula of three phase power.
    (i) In case of star connection (ii) In case of delta connection
6) What is meant by phase sequence? How phase sequence is determined by phase sequence indicator, explain?

7) How many methods are there to measure the power in three phase circuits? Explain any one method.
8) Make a circuit diagram to measure the power with three wattmeter method in three phase star connection and delta connection. And describe the method to find the total power.
9) Which are methods to measure the reactive power in the phase circuit? Explain anyone.

## PART-D

Solve the following problems.

1. Three loads each having resistance 30 Ohm, are connected in star to 415 V, three phase supply, Determine,
   (a) The system phase voltage (b) The phase current
   (c) The line current.
   (a) 240 V (b) 8 A (c) 8 A
2. Three identical coils each of resistance 30 Ohm and inductance 127.3 mH are connected in delta to a 240 V, 50 Hz three phase supply. Determine,
   (a) The phase current (b) the line current.
   (a) 8.8 A (b) 15.24 A
3. Three identical capacitors are connected in delta to a 415 V, 50 Hz three phase supply. If the line current is 15 A Determine, the capacitance of each capacitor?
   66.43 µF
4. Three coils each having resistance 3 Ohm and inductive reactance 4 Ohm are connected (i) In Star (ii) In Delta 415 v three phase supply. Calculate for each connection.
   (a) The lien and phase voltages (b) The phase and line current
5. Three 12 Ohm resistors are connected in Star to 415 v three phase supply. Determine the total power dissipated by the resistor?
   14.4 KW
6. The input power to a A.C motor is measured as 5 KW. If the voltage and current to the motor are 400 v and 8.6 A respectively, Determine the power factor of the system?
   0.839
7. Three coils each having a resistance of 10 ohm and an inductance of 42mH Henry, are connected (a) in star (b) in delta to a three phase, 415v, 50Hz supply. Calculate for each case the line current & the total power absorbed.
8. A 415 v three phase A.C motor has a power output of 12.75 KW and operates a power factor of 0.77 lagging and with an efficiency of 85%. If motor is Delta connected Determine,

(a) The power input (b) The line current (c) The phase current.
15 KW, 27.10 A, 15.65 A

9. Two-watt meters connected to a three-phase motor indicate the total power input to be 12 KW. The power factor is 0.6. Determine the reading of each watt meter.
   $W_1$ = 10.56 kW, $W_2$ = 1.44kW
10. Two-watt meters indicate 10 KW and 3 KW respectively when connected to measure the input power to a three-phase balanced load the reverse switch is being operated on the meter the 3 KW reading. Determine:
    (a) The input power (b) the load power factor
11. A single-phase inductive load is consuming 2 KW active power and 1.5 KVAR Reactive power. Calculate power factor of load.
    Power Factor of load = 0.8 Lagging